

# Package ‘survkl’

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**Title** Estimate Survival Data with Data Integration

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**URL** <https://um-kevinhe.github.io/survkl/>

**Description** Provides flexible and efficient tools for integrating external risk scores into Cox proportional hazards models while accounting for population heterogeneity. Enables robust estimation, improved predictive accuracy, and user-friendly workflows for modern survival analysis. For more information, see Wang et al. (2023) <[doi:10.48550/arXiv.2302.11123](https://doi.org/10.48550/arXiv.2302.11123)>.

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cal_surv_prob	<i>Calculate Survival Probabilities</i>
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### Description

Computes individual survival probabilities from a fitted linear predictor  $z\%*\%beta$  using a stratified Breslow-type baseline hazard estimate.

### Usage

```
cal_surv_prob(z, delta, time, beta, stratum)
```

### Arguments

z	A numeric matrix (or data frame coercible to matrix) of covariates. Each row is an observation and each column a predictor.
delta	A numeric vector of event indicators (1 = event, 0 = censored).
time	A numeric vector of observed times (event or censoring).

beta	A numeric vector of regression coefficients with length equal to the number of columns in z.
stratum	An optional vector specifying the stratum for each observation. If missing, a single-stratum model is assumed.

### Details

Inputs are internally sorted by `stratum` and `time`. Within each stratum, a baseline hazard increment is computed as  $\text{delta}/S_0$ , where  $S_0$  is the risk set sum returned by `ddloglik_S0`. The stratified baseline cumulative hazard  $\text{Lambda}_0$  is then formed by a cumulative sum within stratum, and individual survival curves are computed as  $S(t) = \exp(-\text{Lambda}_0(t) * \exp(z \% \% \text{beta}))$ .

### Value

A numeric matrix of survival probabilities with `nrow(z)` rows and `length(time)` columns. Rows correspond to observations; columns are in the internal sorted order of `(stratum, time)` (i.e., not collapsed to unique event times). Entry  $S[i, j]$  is the estimated survival probability for subject  $i$  evaluated at the  $j$ -th sorted time point.

---

coef.coxkl

*Extract Coefficients from a coxkl Object*

---

### Description

Extracts the estimated regression coefficients (`beta`) from a fitted `coxkl` object. Optionally, a value (or vector) of `eta` can be supplied. If the requested `eta` values are not in the fitted sequence, linear interpolation is performed between the nearest neighboring `eta` values; out-of-range requests error.

### Usage

```
## S3 method for class 'coxkl'
coef(object, eta = NULL, ...)
```

### Arguments

object	An object of class "coxkl", typically the result of <code>coxkl</code> .
eta	Optional numeric value or vector specifying the $\eta$ values for which to extract (or interpolate) coefficients. If <code>NULL</code> , all estimated coefficients are returned.
...	Additional arguments (currently ignored).

### Value

A numeric matrix of regression coefficients. Each column corresponds to one value of `eta`, sorted in ascending order.

## Examples

```
data(ExampleData_lowdim)

train_dat_lowdim <- ExampleData_lowdim$train
beta_external_good_lowdim <- ExampleData_lowdim$beta_external_good
eta_list <- generate_eta(method = "exponential", n = 5, max_eta = 5)

model <- coxkl(z = train_dat_lowdim$z,
              delta = train_dat_lowdim$status,
              time = train_dat_lowdim$time,
              stratum = train_dat_lowdim$stratum,
              beta = beta_external_good_lowdim,
              etas = eta_list)

coef(model)
```

---

coef.coxkl\_enet      *Extract Coefficients from a coxkl\_enet Object*

---

## Description

Extracts the estimated regression coefficients (beta) from a fitted `coxkl_enet` object. Optionally, one or more `lambda` values can be supplied. If requested `lambda` values are not in the fitted sequence, linear interpolation is performed between nearest neighbors; out-of-range requests error.

## Usage

```
## S3 method for class 'coxkl_enet'
coef(object, lambda = NULL, ...)
```

## Arguments

<code>object</code>	An object of class " <code>coxkl_enet</code> ", typically the result of <a href="#">coxkl_enet</a> .
<code>lambda</code>	Optional numeric value or vector specifying the regularization parameter(s) for which to extract (or interpolate) coefficients. If <code>NULL</code> , all estimated coefficients are returned.
<code>...</code>	Additional arguments (currently ignored).

## Value

A numeric matrix of regression coefficients; each column corresponds to one value of `lambda`, sorted in *descending* order.

**Examples**

```

data(ExampleData_highdim)

train_dat_highdim <- ExampleData_highdim$train
beta_external_highdim <- ExampleData_highdim$beta_external

enet_model <- coxkl_enet(z = train_dat_highdim$z,
                        delta = train_dat_highdim$status,
                        time = train_dat_highdim$time,
                        beta = beta_external_highdim,
                        eta = 1,
                        alpha = 1.0)
coef(enet_model)[1:5, 1:10]

```

---

coef.coxkl\_ridge      *Extract Coefficients from a coxkl\_ridge Object*

---

**Description**

Extracts the estimated regression coefficients (beta) from a fitted `coxkl_ridge` object. Optionally, one or more `lambda` values can be supplied. If requested `lambda` values are not in the fitted sequence, linear interpolation is performed between nearest neighbors; out-of-range requests error.

**Usage**

```

## S3 method for class 'coxkl_ridge'
coef(object, lambda = NULL, ...)

```

**Arguments**

<code>object</code>	An object of class "coxkl_ridge", typically the result of <code>coxkl_ridge</code> .
<code>lambda</code>	Optional numeric value or vector specifying the regularization parameter(s) for which to extract (or interpolate) coefficients. If <code>NULL</code> , all estimated coefficients are returned.
<code>...</code>	Additional arguments (currently ignored).

**Value**

A numeric matrix of regression coefficients. Each column corresponds to one value of `lambda`, sorted in *descending* order.

**Examples**

```

data(ExampleData_highdim)

train_dat_highdim <- ExampleData_highdim$train
beta_external_highdim <- ExampleData_highdim$beta_external

model_ridge <- coxkl_ridge(z = train_dat_highdim$,
                          delta = train_dat_highdim$status,
                          time = train_dat_highdim$time,
                          beta = beta_external_highdim,
                          eta = 1)

coef(model_ridge)[1:5, 1:10]

```

---

coxkl	<i>Cox Proportional Hazards Model with KL Divergence for Data Integration</i>
-------	---

---

**Description**

Fits a Cox proportional hazards model that incorporates external information via a Kullback–Leibler (KL) divergence penalty. External information can be supplied either as external risk scores (RS) or as external coefficients (beta). The tuning parameter(s) etas control the strength of integration.

**Usage**

```

coxkl(
  z,
  delta,
  time,
  stratum = NULL,
  RS = NULL,
  beta = NULL,
  etas,
  tol = 1e-04,
  Mstop = 100,
  backtrack = FALSE,
  message = FALSE,
  data_sorted = FALSE,
  beta_initial = NULL
)

```

**Arguments**

z	Numeric matrix of covariates with rows representing observations and columns representing predictor variables. All covariates must be numeric.
delta	Numeric vector of event indicators (1 = event, 0 = censored).

time	Numeric vector of observed event or censoring times. No sorting required.
stratum	Optional numeric or factor vector defining strata.
RS	Optional numeric vector or matrix of external risk scores. Length (or number of rows) must equal the number of observations. If not supplied, beta must be provided.
beta	Optional numeric vector of external coefficients (e.g., from prior studies). Length must equal the number of columns in z. Use zeros to represent covariates without external information. If not supplied, RS must be provided.
etas	Numeric vector of tuning parameters controlling the reliance on external information. Larger values place more weight on the external source.
tol	Convergence tolerance for the optimization algorithm. Default is 1e-4.
Mstop	Maximum number of iterations for the optimization algorithm. Default is 100.
backtrack	Logical; if TRUE, backtracking line search is applied during optimization. Default is FALSE.
message	Logical; if TRUE, progress messages are printed during model fitting. Default is FALSE.
data_sorted	Logical; if TRUE, input data are assumed to be already sorted by stratum and time. Default is FALSE.
beta_initial	Optional numeric vector of length p giving the starting value for the first eta. If NULL, a zero vector is used.

## Details

If beta is supplied (length  $\text{ncol}(z)$ ), external risk scores are computed internally as  $RS = z \%*\% \text{beta}$ . If RS is supplied, it is used directly. Data are optionally sorted by stratum (or a single stratum if NULL) and increasing time when `data_sorted = FALSE`. Estimation proceeds over the sorted data, and the returned `linear.predictors` are mapped back to the original order. Optimization uses warm starts across the (ascending) `etas` grid and supports backtracking line search when `backtrack = TRUE`.

Internally, the routine computes a stratum-wise adjusted event indicator (`delta_tilde`) and maximizes a KL-regularized partial likelihood. The current implementation fixes  $\lambda = 0$  in the low-level optimizer and exposes `etas` as the primary tuning control.

## Value

An object of class "coxkl" containing:

- `eta`: the fitted  $\eta$  sequence.
- `beta`: estimated coefficient matrix ( $p \times |\eta|$ ).
- `linear.predictors`: matrix of linear predictors.
- `likelihood`: vector of partial likelihoods.
- `data`: a list containing the input data used in fitting (`z`, `time`, `delta`, `stratum`, `data_sorted`).

**Examples**

```

data(ExampleData_lowdim)

train_dat_lowdim <- ExampleData_lowdim$train
beta_external_good_lowdim <- ExampleData_lowdim$beta_external_good
eta_list <- generate_eta(method = "exponential", n = 10, max_eta = 5)

model <- coxkl(z = train_dat_lowdim$z,
              delta = train_dat_lowdim$status,
              time = train_dat_lowdim$time,
              stratum = train_dat_lowdim$stratum,
              beta = beta_external_good_lowdim,
              etas = eta_list)

```

---

coxkl\_enet

*Cox Proportional Hazards Model with KL Divergence for Data Integration and Lasso & Elastic Net Penalty*


---

**Description**

Fits a Cox proportional hazards model that incorporates external information using Kullback–Leibler (KL) divergence, with an optional L1 (Lasso) or elastic net penalty on the coefficients. External information can be supplied either as precomputed external risk scores (RS) or as externally derived coefficients (beta). The integration strength is controlled by the tuning parameter eta.

**Usage**

```

coxkl_enet(
  z,
  delta,
  time,
  stratum = NULL,
  RS = NULL,
  beta = NULL,
  eta = NULL,
  alpha = NULL,
  lambda = NULL,
  nlambda = 100,
  lambda.min.ratio = ifelse(n < p, 0.05, 0.001),
  lambda.early.stop = FALSE,
  tol = 1e-04,
  Mstop = 1000,
  max.total.iter = (Mstop * nlambda),
  group = 1:ncol(z),
  group.multiplier = NULL,
  standardize = TRUE,

```

```

nvar.max = ncol(z),
group.max = length(unique(group)),
stop.loss.ratio = 0.001,
actSet = TRUE,
actIter = Mstop,
actGroupNum = sum(unique(group) != 0),
actSetRemove = FALSE,
returnX = FALSE,
trace.lambda = FALSE,
message = FALSE,
data_sorted = FALSE,
...
)

```

### Arguments

<code>z</code>	Numeric matrix of covariates with rows representing observations and columns representing predictor variables. All covariates must be numeric.
<code>delta</code>	Numeric vector of event indicators (1 = event, 0 = censored).
<code>time</code>	Numeric vector of observed event or censoring times. No sorting required.
<code>stratum</code>	Optional numeric or factor vector defining strata.
<code>RS</code>	Optional numeric vector or matrix of external risk scores. Length (or number of rows) must equal the number of observations. If not supplied, beta must be provided.
<code>beta</code>	Optional numeric vector of external coefficients (e.g., from prior studies). Length must equal the number of columns in <code>z</code> . Use zeros to represent covariates without external information. If not supplied, RS must be provided.
<code>eta</code>	Numeric tuning parameter controlling the reliance on external information. Larger values place more weight on the external source.
<code>alpha</code>	Elastic-net mixing parameter in $(0, 1]$ . When $\alpha = 1$ the penalty is lasso.
<code>lambda</code>	Optional nonnegative penalty parameter(s). If a numeric vector is supplied, the path is taken as-is. If NULL, a sequence is generated using <code>nlambda</code> and <code>lambda.min.ratio</code> .
<code>nlambda</code>	Integer number of lambda values to generate when lambda is NULL. Default 100.
<code>lambda.min.ratio</code>	Ratio of the smallest to the largest lambda when generating a sequence (when lambda is NULL). Default 1e-3.
<code>lambda.early.stop</code>	Logical; if TRUE, stop traversing the lambda path early based on convergence or screening criteria. Default FALSE.
<code>tol</code>	Convergence tolerance for the optimization algorithm. Default is 1e-3.
<code>Mstop</code>	Maximum number of iterations for the inner optimization at a given lambda. Default is 1000.
<code>max.total.iter</code>	Maximum total iterations across the entire lambda path. Default is $(Mstop * nlambda)$ .

<code>group</code>	Integer vector of group indices defining group membership of predictors for grouped penalties; use 0 to indicate unpenalized variables.
<code>group.multiplier</code>	A vector of values representing multiplicative factors by which each covariate's penalty is to be multiplied. Default is a vector of 1's.
<code>standardize</code>	Logical; if TRUE, columns of <code>z</code> are standardized prior to fitting, with coefficients re-scaled on output. Default TRUE.
<code>nvar.max</code>	Integer cap on the number of active variables allowed during fitting. Default number of predictors.
<code>group.max</code>	Integer cap on the number of active groups allowed during fitting. Default total number of groups.
<code>stop.loss.ratio</code>	Relative improvement threshold for early stopping along the path; optimization may stop if objective gain falls below this value. Default 1e-3.
<code>actSet</code>	Logical; if TRUE, use an active-set strategy. Default TRUE.
<code>actIter</code>	Maximum number of active-set refinement iterations per lambda. Default <code>Mstop</code> .
<code>actGroupNum</code>	Maximum number of active groups allowed under the active-set scheme.
<code>actSetRemove</code>	Logical; if TRUE, allow dropping variables/groups from the active set during iterations. Default FALSE.
<code>returnX</code>	Logical; if TRUE, return standardized design and related internals in <code>result\$returnX</code> . Default FALSE.
<code>trace.lambda</code>	Logical; if TRUE, record path-wise traces across the lambda sequence. Default FALSE.
<code>message</code>	Logical; if TRUE, progress messages are printed during model fitting. Default is FALSE.
<code>data_sorted</code>	Logical; if TRUE, input is assumed already sorted by <code>stratum</code> then <code>time</code> . Default FALSE.
<code>...</code>	Additional arguments.

### Details

Setting `lambda = 0` reduces to the unpenalized `coxkl` model.

When `lambda > 0`, the model fits a KL-regularized Cox objective with an elastic-net penalty:

$$\ell_{\text{KL}}(\beta; \eta) - \lambda \left\{ \alpha \|\beta\|_1 + (1 - \alpha) \frac{1}{2} \|\beta\|_2^2 \right\},$$

where  $\alpha = 1$  gives lasso and  $0 < \alpha < 1$  gives elastic net. Grouped penalties are supported via `group` (use 0 for unpenalized variables), with optional per-group scaling through `group.multiplier`. If `lambda` is NULL, a decreasing path of length `nlambda` is generated using `lambda.min.ratio`; early stopping can prune the path (`lambda.early.stop`, `stop.loss.ratio`). When `standardize = TRUE`, predictors are standardized for fitting and coefficients are rescaled on output. If `data_sorted = FALSE`, data are sorted by `stratum` then `time` for optimization and predictions are returned in the original order (reported via `W = exp(linear predictors)`). An active-set scheme (`actSet`, `actIter`, `nvar.max`, `group.max`, `actGroupNum`, `actSetRemove`) is used to accelerate the solution along the lambda path.

**Value**

An object of class "coxkl\_enet", a list with components:

beta Coefficient estimates (vector or matrix across the path).  
 group A factor of the original group assignments.  
 lambda The lambda value(s) used or generated.  
 alpha The elastic-net mixing parameter used.  
 likelihood Vector of log-partial likelihoods for each lambda.  
 n Number of observations.  
 df Effective degrees of freedom (e.g., number of nonzero coefficients or group-adjusted count) along the path.  
 iter Number of iterations taken (per lambda and/or total).  
 W Exponentiated linear predictors on the original scale.  
 group.multiplier Group-specific penalty multipliers used.  
 returnX Only when returnX = TRUE: a list with elements XX (standardization/orthogonalization info from std.Z), time, delta, stratum, and RS.

**See Also**

[coxkl](#)

**Examples**

```
data(ExampleData_highdim)

train_dat_highdim <- ExampleData_highdim$train
beta_external_highdim <- ExampleData_highdim$beta_external

model_enet <- coxkl_enet(z = train_dat_highdim$z,
                        delta = train_dat_highdim$status,
                        time = train_dat_highdim$time,
                        beta = beta_external_highdim,
                        eta = 0,
                        alpha = 1.0)
```

---

coxkl\_ridge

*Cox Proportional Hazards Model with Ridge Penalty and External Information*

---

**Description**

Fits a Cox proportional hazards model using a ridge-type penalty (L2) on all covariates. The model can integrate external information either as precomputed risk scores (RS) or externally supplied coefficients (beta). A tuning parameter eta controls the relative weight of the external information. If lambda is not provided, a lambda sequence is automatically generated.

**Usage**

```

coxkl_ridge(
  z,
  delta,
  time,
  stratum = NULL,
  RS = NULL,
  beta = NULL,
  eta = NULL,
  lambda = NULL,
  nlambda = 100,
  lambda.min.ratio = ifelse(n_obs < n_vars, 0.01, 1e-04),
  penalty.factor = 0.999,
  tol = 1e-04,
  Mstop = 50,
  backtrack = FALSE,
  message = FALSE,
  data_sorted = FALSE,
  beta_initial = NULL,
  ...
)

```

**Arguments**

<code>z</code>	Numeric matrix of covariates (observations in rows, predictors in columns).
<code>delta</code>	Numeric vector of event indicators (1 = event, 0 = censored).
<code>time</code>	Numeric vector of observed times.
<code>stratum</code>	Optional numeric or factor vector specifying strata.
<code>RS</code>	Optional numeric vector or matrix of external risk scores.
<code>beta</code>	Optional numeric vector of externally derived coefficients.
<code>eta</code>	Non-negative scalar controlling the strength of external information.
<code>lambda</code>	Optional numeric scalar or vector of penalty parameters. If NULL, a sequence is generated automatically.
<code>nlambda</code>	Number of lambda values to generate if lambda is NULL.
<code>lambda.min.ratio</code>	Ratio defining the minimum lambda relative to lambda.max.
<code>penalty.factor</code>	Numeric scalar in $[0, 1)$ . Controls the overall strength of the penalty when generating the ridge regression lambda sequence. Smaller values correspond to stronger penalization. Only used when lambda = NULL.
<code>tol</code>	Convergence tolerance for the iterative estimation algorithm.
<code>Mstop</code>	Maximum number of iterations for estimation.
<code>backtrack</code>	Logical; if TRUE, uses backtracking line search.
<code>message</code>	Logical; if TRUE, progress messages are printed during model fitting. Default is FALSE.

<code>data_sorted</code>	Logical; if TRUE, assumes input data is already sorted by strata and time.
<code>beta_initial</code>	Optional; default NULL. When NULL, the algorithm initializes <code>beta_initial</code> to a zero vector as a warm start
<code>...</code>	Additional arguments.

### Details

The estimator maximizes a KL-regularized Cox partial log-likelihood with a ridge (L2) penalty on all coefficients. External information is incorporated via a KL term weighted by `eta`: if `beta` is supplied (length `ncol(z)`), external risk scores are computed internally as `RS = z %*% beta`; otherwise `RS` must be provided. If `lambda` is NULL, a decreasing `lambda` path of length `nlambda` is generated using `lambda.min.ratio` (its overall scale is influenced by `penalty.factor`). Optimization proceeds along the `lambda` path with warm starts (re-using the previous solution as `beta_initial`); when `beta_initial = NULL`, the first step uses zeros. If `data_sorted = FALSE`, data are sorted by `stratum` and `time` for fitting and the returned linear predictors are mapped back to the original observation order. `tol`, `Mstop`, and `backtrack` control convergence and line search.

### Value

An object of class "coxkl\_ridge" containing:

- `lambda`: The `lambda` sequence used for estimation.
- `beta`: Matrix of estimated coefficients for each `lambda`.
- `linear.predictors`: Matrix of linear predictors.
- `likelihood`: Vector of log-partial likelihoods.
- `data`: A list containing the input data used in fitting (`z`, `time`, `delta`, `stratum`, `data_sorted`).

### Examples

```
data(ExampleData_highdim)

train_dat_highdim <- ExampleData_highdim$train
beta_external_highdim <- ExampleData_highdim$beta_external

model_ridge <- coxkl_ridge(z = train_dat_highdim$z,
                          delta = train_dat_highdim$status,
                          time = train_dat_highdim$time,
                          beta = beta_external_highdim)
```

**Description**

Performs K-fold cross-validation to select the integration parameter eta for the Cox-KL model. Each fold fits the model on a training split and evaluates on the held-out split using the specified performance criterion.

**Usage**

```
cv.coxkl(
  z,
  delta,
  time,
  stratum = NULL,
  RS = NULL,
  beta = NULL,
  etas = NULL,
  tol = 1e-04,
  Mstop = 100,
  backtrack = FALSE,
  nfolds = 5,
  criteria = c("V&VH", "LinPred", "CIndex_pooled", "CIndex_foldaverage"),
  c_index_stratum = NULL,
  message = FALSE,
  seed = NULL,
  ...
)
```

**Arguments**

<code>z</code>	Numeric matrix of covariates (rows = observations, columns = variables).
<code>delta</code>	Numeric vector of event indicators (1 = event, 0 = censored).
<code>time</code>	Numeric vector of observed event or censoring times.
<code>stratum</code>	Optional numeric or factor vector defining strata. If NULL, all observations are treated as a single stratum.
<code>RS</code>	Optional numeric vector or matrix of external risk scores. If omitted, beta must be supplied.
<code>beta</code>	Optional numeric vector of external coefficients. If omitted, RS must be supplied.
<code>etas</code>	Numeric vector of candidate tuning values to be cross-validated. (required). Values are internally sorted in ascending order.
<code>tol</code>	Convergence tolerance for the optimizer used inside <code>coxkl</code> . Default 1e-4.
<code>Mstop</code>	Maximum number of Newton iterations used inside <code>coxkl</code> . Default 100.
<code>backtrack</code>	Logical; if TRUE, backtracking line search is applied during optimization. Default is FALSE.
<code>nfolds</code>	Number of cross-validation folds. Default 5.

<code>criteria</code>	Character string specifying the performance criterion. Choices are "V&VH" (default), "LinPred", "CIndex_pooled", or "CIndex_foldaverage".
<code>c_index_stratum</code>	Optional stratum vector. Only required when <code>criteria</code> is set to "CIndex_pooled" or "CIndex_foldaverage", and a stratified C-index is desired while the fitted model is non-stratified. Default NULL.
<code>message</code>	Logical; if TRUE, prints progress messages and per-fold progress bars. Default FALSE.
<code>seed</code>	Optional integer seed for reproducible fold assignment. Default NULL.
<code>...</code>	Additional arguments passed to <code>coxkl</code> .

### Details

External information is required: supply either RS or beta (if beta is given, RS is computed as  $z \times \beta$ ). Folds are created with stratification by `stratum` and censoring status. Within each fold and each candidate eta, the function fits `coxkl` on the training split with warm-starts initialized to zero and evaluates on the test split:

- "V&VH": uses the difference of partial log-likelihoods between full and training fits; reported as  $-2$  times the aggregated quantity.
- "LinPred": aggregates the test-split linear predictors across folds and evaluates  $-2$  times the partial log-likelihood on the full data.
- "CIndex\_pooled": pools pairwise comparable counts across folds (numerator/denominator).
- "CIndex\_foldaverage": averages the per-fold stratified C-index.

The function also computes an external baseline statistic from RS using the same criterion for comparison.

### Value

An object of class "cv.coxkl" with components:

`internal_stat` A data.frame with one row per eta containing eta and the cross-validated measure named according to `criteria` (one of `V&VH_Loss`, `LinPred_Loss`, `CIndex_pooled`, `CIndex_foldaverage`).

`external_stat` Scalar baseline statistic computed from RS under the same `criteria`.

`criteria` The evaluation criterion used.

`nfolds` Number of folds.

### Examples

```
data(ExampleData_lowdim)

train_dat_lowdim <- ExampleData_lowdim$train
beta_external_good_lowdim <- ExampleData_lowdim$beta_external_good

etas <- generate_eta(method = "exponential", n = 10, max_eta = 5)

cv_res <- cv.coxkl(z = train_dat_lowdim$,
```

```

delta = train_dat_lowdim$status,
time = train_dat_lowdim$time,
beta = beta_external_good_lowdim,
etas = etas)

```

---

cv.coxkl\_enet

---

*Cross-Validation for CoxKL Model with elastic net & lasso penalty*


---

## Description

This function performs cross-validation on the high-dimensional Cox model with Kullback–Leibler (KL) penalty. It tunes the parameter  $\eta$  (external information weight) using user-specified cross-validation criteria, while also evaluating a  $\lambda$  path (either provided or generated) and selecting the best  $\lambda$  per  $\eta$ .

## Usage

```

cv.coxkl_enet(
  z,
  delta,
  time,
  stratum = NULL,
  RS = NULL,
  beta = NULL,
  etas,
  alpha = 1,
  lambda = NULL,
  nlambda = 100,
  lambda.min.ratio = ifelse(n < p, 0.05, 0.001),
  nfolds = 5,
  cv.criteria = c("V&VH", "LinPred", "CIndex_pooled", "CIndex_foldaverage"),
  c_index_stratum = NULL,
  message = FALSE,
  seed = NULL,
  ...
)

```

## Arguments

<code>z</code>	Numeric matrix of covariates with rows representing individuals and columns representing predictors.
<code>delta</code>	Numeric vector of event indicators (1 = event, 0 = censored).
<code>time</code>	Numeric vector of observed times (event or censoring).
<code>stratum</code>	Optional factor or numeric vector indicating strata.
<code>RS</code>	Optional numeric vector or matrix of external risk scores. If not provided, $\beta$ must be supplied.

beta	Optional numeric vector of external coefficients (length equal to <code>ncol(z)</code> ). If not provided, RS must be supplied.
etas	Numeric vector of candidate eta values to be evaluated.
alpha	Elastic-net mixing parameter in $(0, 1]$ . Default = 1 (lasso penalty).
lambda	Optional numeric scalar or vector of penalty parameters. If NULL, a decreasing path is generated using <code>nlambda</code> and <code>lambda.min.ratio</code> .
nlambda	Integer number of lambda values to generate when lambda is NULL. Default 100.
lambda.min.ratio	Ratio of the smallest to the largest lambda when generating a sequence (when lambda is NULL). Default 0.05 when $n < p$ , otherwise 1e-3.
nfolds	Integer; number of cross-validation folds. Default = 5.
cv.criteria	Character string specifying the cross-validation criterion. Choices are: <ul style="list-style-type: none"> <li>• "V&amp;VH" (default): "V&amp;VH" loss.</li> <li>• "LinPred": loss based on cross-validated linear predictors.</li> <li>• "CIndex_pooled": pool all held-out predictions and compute one overall C-index.</li> <li>• "CIndex_foldaverage": average C-index across folds.</li> </ul>
c_index_stratum	Optional stratum vector. Used only when <code>cv.criteria</code> is "CIndex_pooled" or "CIndex_foldaverage" to compute a stratified C-index while the fitted model is non-stratified; if supplied, it must be identical to <code>stratum</code> . Default NULL.
message	Logical; whether to print progress messages. Default = FALSE.
seed	Optional integer random seed for fold assignment.
...	Additional arguments passed to <code>coxkl_enet</code> .

## Details

Data are sorted by `stratum` and `time`. External info must be from RS or `beta` (if `beta` given with length `ncol(z)`, `RS = z %>% beta`);  $\alpha \in (0, 1]$ .

For each candidate `eta`, a decreasing `lambda` path is used (generated from `nlambda/lambda.min.ratio` if `lambda = NULL`); CV folds are created by `get_fold`. Each fold fits `coxkl_enet` on the training split (full `lambda` path) and evaluates the chosen criterion on the test split.

Aggregation follows the code paths for "V&VH", "LinPred", "CIndex\_pooled", or "CIndex\_foldaverage":

- "V&VH": sums `pl(full) - pl(train)` across folds (reported as loss via `Loss = -2 * score`).
- "LinPred": aggregates test-fold linear predictors and evaluates partial log-likelihood on full data (reported as `Loss = -2 * score`).
- "CIndex\_pooled": pools comparable-pair numerators/denominators across folds to compute one C-index.
- "CIndex\_foldaverage": averages the per-fold stratified C-index.

The best `lambda` is selected per `eta` (min loss / max C-index), and the function returns full results, the per-eta optimum, corresponding coefficients, and an external baseline from RS.

**Value**

An object of class "cv.coxkl\_enet":

`integrated_stat.full_results` Data frame with columns `eta`, `lambda`, and the aggregated CV score for each `lambda` under the chosen `cv.criteria`. For loss criteria, an additional column with the transformed loss ( $\text{Loss} = -2 * \text{score}$ ); for C-index criteria, a column named `CIndex_pooled` or `CIndex_foldaverage`.

`integrated_stat.best_per_eta` Data frame with the best `lambda` (per `eta`) according to the chosen `cv.criteria` (minimizing loss or maximizing C-index).

`integrated_stat.betahat_best` Matrix of coefficient vectors (columns) corresponding to the best `lambda` for each `eta`.

`external_stat` Scalar baseline statistic computed from the external risk score `RS` under the same `cv.criteria`.

`criteria` The evaluation criterion used (as provided in `cv.criteria`).

`alpha` The elastic-net mixing parameter used.

`nfolds` Number of folds.

**Examples**

```
data(ExampleData_highdim)

train_dat_highdim <- ExampleData_highdim$train
beta_external_highdim <- ExampleData_highdim$beta_external

etas <- generate_eta(method = "exponential", n = 10, max_eta = 100)

cv_res <- cv.coxkl_enet(z = train_dat_highdim$z,
                      delta = train_dat_highdim$status,
                      time = train_dat_highdim$time,
                      stratum = NULL,
                      RS = NULL,
                      beta = beta_external_highdim,
                      etas = etas,
                      alpha = 1.0)
```

---

 cv.coxkl\_ridge

---

*Cross-Validation for CoxKL Ridge Model (eta tuning)*


---

**Description**

This function performs cross-validation on the Cox model with Kullback–Leibler (KL) penalty and ridge (L2) regularization. It tunes the parameter `eta` (external information weight) using user-specified cross-validation criteria, while internally evaluating a `lambda` path (provided or generated) and selecting the best `lambda` per `eta`.

**Usage**

```

cv.coxkl_ridge(
  z,
  delta,
  time,
  stratum = NULL,
  RS = NULL,
  beta = NULL,
  etas,
  lambda = NULL,
  nlambda = 100,
  lambda.min.ratio = ifelse(n_obs < n_vars, 0.01, 1e-04),
  nfolds = 5,
  cv.criteria = c("V&VH", "LinPred", "CIndex_pooled", "CIndex_foldaverage"),
  c_index_stratum = NULL,
  message = FALSE,
  seed = NULL,
  ...
)

```

**Arguments**

<code>z</code>	Numeric matrix of covariates with rows representing individuals and columns representing predictors.
<code>delta</code>	Numeric vector of event indicators (1 = event, 0 = censored).
<code>time</code>	Numeric vector of observed times (event or censoring).
<code>stratum</code>	Optional factor or numeric vector indicating strata.
<code>RS</code>	Optional numeric vector or matrix of external risk scores. If not provided, beta must be supplied.
<code>beta</code>	Optional numeric vector of external coefficients (length equal to <code>ncol(z)</code> ). If not provided, RS must be supplied.
<code>etas</code>	Numeric vector of candidate eta values to be evaluated.
<code>lambda</code>	Optional numeric scalar or vector of penalty parameters. If NULL, a sequence is generated automatically.
<code>nlambda</code>	Integer number of lambda values to generate when lambda is NULL. Default 100.
<code>lambda.min.ratio</code>	Ratio of the smallest to the largest lambda when generating a sequence (when lambda is NULL). Default 0.01 when $n < p$ , otherwise 1e-4.
<code>nfolds</code>	Integer; number of cross-validation folds. Default 5.
<code>cv.criteria</code>	Character string specifying the cross-validation criterion. Choices are: <ul style="list-style-type: none"> <li>• "V&amp;VH" (default): "V&amp;VH" loss.</li> <li>• "LinPred": loss based on cross-validated linear predictors.</li> <li>• "CIndex_pooled": pool all held-out predictions and compute one overall C-index.</li> </ul>

	<ul style="list-style-type: none"> <li>• "CIndex_foldaverage": average C-index across folds.</li> </ul>
c_index_stratum	Optional stratum vector. Used only when cv.criteria is "CIndex_pooled" or "CIndex_foldaverage" to compute a stratified C-index for a non-stratified fit; if supplied, it must be identical to stratum. Default NULL.
message	Logical; whether to print progress messages. Default FALSE.
seed	Optional integer random seed for fold assignment.
...	Additional arguments passed to <code>coxkl_ridge</code> .

## Details

Data are sorted by stratum and time. External information must be given via RS or beta (if beta has length `ncol(z)`, the function computes  $RS = z \%*\% beta$ ). For each candidate eta, a lambda path is determined (generated if `lambda = NULL`, otherwise the supplied lambda values are sorted decreasingly). Cross-validation folds are created by `get_fold`. In each fold, `coxkl_ridge` is fit on the training split across the full lambda path with `data_sorted = TRUE`, and the chosen criterion is evaluated on the test split and aggregated:

- "V&VH": sums  $pl(full) - pl(train)$  across folds (reported as loss via  $Loss = -2 * score$ ).
- "LinPred": aggregates test-fold linear predictors and evaluates partial log-likelihood on full data (reported as  $Loss = -2 * score$ ).
- "CIndex\_pooled": pools comparable-pair numerators/denominators across folds to compute one C-index.
- "CIndex\_foldaverage": averages the per-fold stratified C-index.

The best lambda is chosen per eta (minimizing loss or maximizing C-index). The function also computes an external baseline statistic from RS under the same criterion.

## Value

An object of class "cv.coxkl\_ridge":

`integrated_stat.full_results` Data frame with columns eta, lambda, and the aggregated CV score per lambda; for loss criteria an additional column  $Loss = -2 * score$ ; for C-index criteria a column named `CIndex_pooled` or `CIndex_foldaverage`.

`integrated_stat.best_per_eta` Data frame with the best lambda (per eta) according to the chosen criterion.

`external_stat` Scalar baseline statistic computed from RS under the same cv.criteria.

`criteria` The evaluation criterion used.

`nfolds` Number of folds.

## Examples

```
data(ExampleData_highdim)
```

```
train_dat_highdim <- ExampleData_highdim$strain
```

```
beta_external_highdim <- ExampleData_highdim$beta_external
```

```
etas <- generate_eta(method = "exponential", n = 10, max_eta = 100)
cv_res <- cv.coxkl_ridge(z = train_dat_highdim$,
                        delta = train_dat_highdim$status,
                        time = train_dat_highdim$time,
                        beta = beta_external_highdim,
                        etas = etas)
```

---

cv.plot

*Plot Cross-Validation Results vs Eta*


---

### Description

Plots cross-validated performance across  $\eta$  for `cv.coxkl`, `cv.coxkl_ridge`, or `cv.coxkl_enet` results. The main CV curve is drawn as a solid purple line; a green dotted horizontal reference line is placed at the value corresponding to  $\eta = 0$  (or the closest available  $\eta$ ), with a solid green point marking that reference level.

### Usage

```
cv.plot(object, line_color = "#7570B3", baseline_color = "#1B9E77", ...)
```

### Arguments

<code>object</code>	A fitted cross-validation result of class <code>"cv.coxkl"</code> , <code>"cv.coxkl_ridge"</code> , or <code>"cv.coxkl_enet"</code> .
<code>line_color</code>	Color for the CV performance curve. Default <code>"#7570B3"</code> .
<code>baseline_color</code>	Color for the horizontal reference line and point. Default <code>"#1B9E77"</code> .
<code>...</code>	Additional arguments (currently ignored).

### Details

The function reads the performance metric from the object:

- For `"cv.coxkl"`: uses `object$internal_stat` (one row per  $\eta$ ).
- For `"cv.coxkl_ridge"` and `"cv.coxkl_enet"`: uses `object$integrated_stat.best_per_eta` (best  $\lambda$  per  $\eta$ ).

The y-axis label is set to "Loss" if criteria in the object is "V&VH" or "LinPred"; otherwise it is "C Index". The horizontal reference ("baseline") is taken from the plotted series at  $\eta = 0$  (or the nearest  $\eta$  present in the results).

### Value

A ggplot object showing cross-validation performance versus  $\eta$ .

**See Also**

[cv.coxkl](#), [cv.coxkl\\_ridge](#), [cv.coxkl\\_enet](#)

**Examples**

```
data(ExampleData_lowdim)

train_dat_lowdim <- ExampleData_lowdim$train
beta_external_good_lowdim <- ExampleData_lowdim$beta_external_good

etas <- generate_eta(method = "exponential", n = 100, max_eta = 30)
cv_res <- cv.coxkl(z = train_dat_lowdim$z,
                  delta = train_dat_lowdim$status,
                  time = train_dat_lowdim$time,
                  stratum = train_dat_lowdim$stratum,
                  beta = beta_external_good_lowdim,
                  etas = etas,
                  nfolds = 5,
                  criteria = c("V&VH"),
                  seed = 1)

cv.plot(cv_res)
```

---

ExampleData\_highdim    *Example high-dimensional survival data*

---

**Description**

A simulated survival dataset in a high-dimensional linear setting with 50 covariates (6 signals + 44 AR(1) noise), Weibull baseline hazard, and controlled censoring. Includes internal train/test sets, and an external-data-estimated coefficient vector.

**Usage**

```
data(ExampleData_highdim)
```

**Format**

A list containing the following elements:

**train** A list with components:

**z** Data frame of size  $n_{\text{train}} \times 50$  with covariates Z1–Z50.

**status** Vector of event indicators (1=event, 0=censored).

**time** Numeric vector of observed times  $\min(T, C)$ .

**stratum** Vector of stratum labels (here all 1).

**test** A list with the same structure as **train**, with size  $n_{\text{test}} \times 50$  for **z**.

**beta\_external** Numeric vector (length 50, named Z1–Z50) of Cox coefficients estimated on an external dataset using only Z1–Z6 and expanded to length 50 (zeros for Z7–Z50).

**Details**

Data-generating mechanism:

- Covariates: 50 variables with signals Z1–Z6 and noise Z7–Z50.
  - Z1, Z2 ~ bivariate normal with AR(1) correlation  $\rho = 0.5$ .
  - Z3, Z4 ~ independent Bernoulli(0.5).
  - Z5 ~  $N(2, 1)$ , Z6 ~  $N(-2, 1)$  (group indicator fixed at 1).
  - Z7–Z50 ~ multivariate normal with AR(1) correlation  $\rho = 0.5$ .
- True coefficients:  $\beta = (0.3, -0.3, 0.3, -0.3, 0.3, -0.3, 0, \dots, 0)$  (length 50).
- Event times: Weibull baseline hazard  $h_0(t) = \lambda \nu t^{\nu-1}$  with  $\lambda = 1$ ,  $\nu = 2$ . Given linear predictor  $\eta = Z^\top \beta$ , draw  $U \sim \text{Unif}(0, 1)$  and set

$$T = \left( \frac{-\log U}{\lambda e^\eta} \right)^{1/\nu}.$$

- Censoring:  $C \sim \text{Unif}(0, \text{ub})$  with ub tuned iteratively to achieve the target censoring rate (internal: 0.70; external: 0.50). Observed time is  $\min(T, C)$ , status is  $\mathbf{1}\{T \leq C\}$ .
- External coefficients: Fit a Cox model  $\text{Surv}(\text{time}, \text{status}) \sim Z1 + \dots + Z6$  on the external data (Breslow ties), then place the estimated coefficients into a length-50 vector (zeros elsewhere).

**Examples**

```
data(ExampleData_highdim)

head(ExampleData_highdim$train$z)
table(ExampleData_highdim$train$status)
summary(ExampleData_highdim$train$time)

head(ExampleData_highdim$test$z)
table(ExampleData_highdim$test$status)
summary(ExampleData_highdim$test$time)
```

---

ExampleData\_lowdim      *Example low-dimensional survival data*

---

**Description**

A simulated survival dataset in a low-dimensional linear setting with 6 covariates (2 correlated continuous, 2 binary, 2 mean-shifted normals), Weibull baseline hazard, and controlled censoring. Includes internal train/test sets, and three external-quality coefficient vectors.

**Usage**

```
data(ExampleData_lowdim)
```

**Format**

A list containing the following elements:

**train** A list with components:

**z** Data frame of size  $n_{\text{train}} \times 6$  with covariates Z1–Z6.

**status** Vector of event indicators (1=event, 0=censored).

**time** Numeric vector of observed times  $\min(T, C)$ .

**stratum** Vector of stratum labels (here all 1).

**test** A list with the same structure as **train**, with size  $n_{\text{test}} \times 6$  for **z**.

**beta\_external\_good** Numeric vector (length 6; named Z1–Z6) of Cox coefficients estimated on a "Good" external dataset using all Z1–Z6.

**beta\_external\_fair** Numeric vector (length 6; names Z1–Z6) of Cox coefficients estimated on a "Fair" external dataset using a reduced subset Z1, Z3, Z5, Z6; coefficients for variables not used are 0.

**beta\_external\_poor** Numeric vector (length 6; names Z1–Z6) of Cox coefficients estimated on a "Poor" external dataset using Z1 and Z5 only; remaining entries are 0.

**Details**

Data-generating mechanism:

- Covariates: 6 variables Z1–Z6.
  - Z1, Z2 ~ bivariate normal with AR(1) correlation  $\rho = 0.5$ .
  - Z3, Z4 ~ independent Bernoulli(0.5).
  - Z5 ~  $N(2, 1)$ , Z6 ~  $N(-2, 1)$  (group indicator fixed at 1 for internal train/test).
- True coefficients:  $\beta = (0.3, -0.3, 0.3, -0.3, 0.3, -0.3)$  (length 6).
- Event times: Weibull baseline hazard  $h_0(t) = \lambda \nu t^{\nu-1}$  with  $\lambda = 1$ ,  $\nu = 2$ . Given linear predictor  $\eta = Z^\top \beta$ , draw  $U \sim \text{Unif}(0, 1)$  and set

$$T = \left( \frac{-\log U}{\lambda e^\eta} \right)^{1/\nu}.$$

- Censoring:  $C \sim \text{Unif}(0, \text{ub})$  with ub tuned iteratively to achieve the target censoring rate (internal: 0.70; external: 0.50). Observed time is  $\min(T, C)$ , status is  $\mathbf{1}\{T \leq C\}$ .
- External coefficients: For each quality level ("Good", "Fair", "Poor"), fit a Cox model `Surv(time, status) ~ Z1 + ...` on the corresponding external data (Breslow ties) using the specified covariate subset; place estimates into a length-6 vector named Z1–Z6 with zeros for variables not included.

**Examples**

```
data(ExampleData_lowdim)

head(ExampleData_lowdim$train$z)
table(ExampleData_lowdim$train$status)
summary(ExampleData_lowdim$train$time)
```

```
head(ExampleData_lowdim$test$z)
table(ExampleData_lowdim$test$status)
summary(ExampleData_lowdim$test$time)
```

---

generate_eta	<i>Generate a Sequence of Tuning Parameters (eta)</i>
--------------	---

---

### Description

Produces a numeric vector of eta values to be used in Cox–KL model.

### Usage

```
generate_eta(method = "exponential", n = 10, max_eta = 5, min_eta = 0)
```

### Arguments

method	Character string selecting how to generate eta: “linear” or “exponential”. Default is “exponential”. for an exponentially spaced sequence scaled to max_eta. Default is “exponential”.
n	Integer, the number of eta values to generate. Default is 10.
max_eta	Numeric, the maximum value of eta in the sequence. Default is 5.
min_eta	Numeric, the minimum value of eta in the sequence. Default is 0.

### Details

- *Exponential*: values are formed by exponentiating a grid from  $\log(1)$  to  $\log(100)$ , then linearly rescaling to the interval  $[0, \max\_eta]$ . Thus the smallest value equals 0 and the largest equals  $\max\_eta$ .
- *Linear*: the current implementation calls `seq(min_eta, max_eta, length.out = n)` and therefore assumes a numeric object `min_eta` exists in the calling environment.

Only the exact strings “linear” and “exponential” are supported; other values for `method` will result in an error because `eta_values` is never created.

### Value

Numeric vector of length `n` containing the generated eta values.

### Examples

```
# Generate 10 exponentially spaced eta values up to 5
generate_eta(method = "exponential", n = 10, max_eta = 5)

# Generate 5 linearly spaced eta values up to 3
generate_eta(method = "linear", n = 5, max_eta = 3)
```

---

loss_fn	<i>Calculate the Log-Partial Likelihood for a Stratified Cox Model</i>
---------	--

---

### Description

Computes the stratified Cox partial log-likelihood for given covariates, event indicators, times, and coefficients.

### Usage

```
loss_fn(z, delta, time, stratum, beta)
```

### Arguments

z	A numeric matrix (or data frame coercible to matrix) of covariates. Each row is an observation and each column a predictor.
delta	A numeric vector of event indicators (1 = event, 0 = censored).
time	A numeric vector of observed times (event or censoring).
stratum	An optional vector specifying the stratum for each observation (factor/character/numeric). If missing, a single-stratum model is assumed.
beta	A numeric vector of regression coefficients with length equal to the number of columns in z.

### Details

Inputs are internally sorted by stratum and time. The function evaluates the stratified Cox partial log-likelihood using the supplied z, delta, beta, and the stratum sizes.

### Value

A single numeric value giving the stratified Cox partial log-likelihood.

---

plot.coxkl	<i>Plot Model Performance vs Eta for coxkl</i>
------------	--

---

### Description

Plots model performance across the eta sequence. Performance is either loss (-2 times partial log-likelihood) or concordance index (C-index). If no test data are provided, the curve is computed on the training data stored in x\$data.

**Usage**

```
## S3 method for class 'coxkl'
plot(
  x,
  test_z = NULL,
  test_time = NULL,
  test_delta = NULL,
  test_stratum = NULL,
  criteria = c("loss", "CIndex"),
  ...
)
```

**Arguments**

x	A fitted model object of class "coxkl".
test_z	Optional numeric matrix of test covariates.
test_time	Optional numeric vector of test survival times.
test_delta	Optional numeric vector of test event indicators.
test_stratum	Optional vector of test stratum membership.
criteria	Character string: "loss" or "CIndex".
...	Additional arguments (ignored).

**Details**

When `criteria = "loss"` and no test data are supplied, the plotted values are  $(-2 * x\$likelihood) / n$ , where  $n$  is the number of rows in the (training) data. When test data are provided, performance is computed via `test_eval(..., criteria = "loss")` and divided by the test sample size. For `criteria = "CIndex"`, performance is computed via `test_eval(..., criteria = "CIndex")` on the chosen dataset. The plot adds a dotted horizontal reference line at the value corresponding to  $\eta = 0$  (closest point on the  $\eta$  grid).

**Value**

A ggplot object showing the performance curve.

**Examples**

```
data(ExampleData_lowdim)

train_dat_lowdim <- ExampleData_lowdim$train
test_dat_lowdim <- ExampleData_lowdim$test
beta_external_good_lowdim <- ExampleData_lowdim$beta_external_good
eta_grid <- generate_eta(method = "exponential", n = 100, max_eta = 30)

model <- coxkl(z = train_dat_lowdim$z,
              delta = train_dat_lowdim$status,
              time = train_dat_lowdim$time,
              stratum = train_dat_lowdim$stratum,
```

```

        beta = beta_external_good_lowdim,
        etas = eta_grid)
plot(model,
      test_z = test_dat_lowdim$,
      test_time = test_dat_lowdim$time,
      test_delta = test_dat_lowdim$status,
      test_stratum = test_dat_lowdim$stratum,
      criteria = "loss")

```

---

plot.coxkl\_enet

*Plot Model Performance vs Lambda for coxkl\_enet*


---

### Description

Plots model performance across the lambda sequence. Performance is loss (-2 times partial log-likelihood) or concordance index (C-index). If no test data are provided, the curve uses the training data stored in `x$data`.

### Usage

```

## S3 method for class 'coxkl_enet'
plot(
  x,
  test_z = NULL,
  test_time = NULL,
  test_delta = NULL,
  test_stratum = NULL,
  criteria = c("loss", "CIndex"),
  ...
)

```

### Arguments

<code>x</code>	A fitted model object of class "coxkl_enet".
<code>test_z</code>	Optional numeric matrix of test covariates.
<code>test_time</code>	Optional numeric vector of test survival times.
<code>test_delta</code>	Optional numeric vector of test event indicators.
<code>test_stratum</code>	Optional vector of test stratum membership.
<code>criteria</code>	Character string: "loss" or "CIndex".
<code>...</code>	Additional arguments (ignored).

### Details

When `criteria = "loss"` and no test data are supplied, the plotted values are  $-2 * x$likelihood$  (no normalization). When test data are provided, performance is computed via `test_eval(..., criteria)`. The x-axis is shown in decreasing lambda with a reversed log10 scale.

**Value**

A ggplot object showing the performance curve.

**Examples**

```
data(ExampleData_highdim)

train_dat_highdim <- ExampleData_highdim$train
test_dat_highdim <- ExampleData_highdim$test
beta_external_highdim <- ExampleData_highdim$beta_external

model_enet <- coxkl_enet(z = train_dat_highdim$z,
                        delta = train_dat_highdim$status,
                        time = train_dat_highdim$time,
                        beta = beta_external_highdim,
                        eta = 1,
                        alpha = 1.0)

plot(model_enet,
     test_z = test_dat_highdim$z,
     test_time = test_dat_highdim$time,
     test_delta = test_dat_highdim$status,
     test_stratum = test_dat_highdim$stratum,
     criteria = "loss")
```

---

plot.coxkl\_ride

*Plot Model Performance vs Lambda for coxkl\_ride*


---

**Description**

Plots model performance across the lambda sequence. Performance is loss ( $-2$  times partial log-likelihood) or concordance index (C-index). If no test data are provided, the curve uses the training data stored in `x$data`.

**Usage**

```
## S3 method for class 'coxkl_ride'
plot(
  x,
  test_z = NULL,
  test_time = NULL,
  test_delta = NULL,
  test_stratum = NULL,
  criteria = c("loss", "CIndex"),
  ...
)
```

**Arguments**

x	A fitted model object of class "coxkl_ridge".
test_z	Optional numeric matrix of test covariates.
test_time	Optional numeric vector of test survival times.
test_delta	Optional numeric vector of test event indicators.
test_stratum	Optional vector of test stratum membership.
criteria	Character string: "loss" or "CIndex".
...	Additional arguments (ignored).

**Details**

When `criteria = "loss"` and no test data are supplied, the plotted values are  $-2 * x\$likelihood$  (no normalization). When test data are provided, performance is computed via `test_eval(..., criteria)`. The x-axis is shown in decreasing lambda with a reversed log10 scale.

**Value**

A ggplot object showing the performance curve.

**Examples**

```
data(ExampleData_highdim)

train_dat_highdim <- ExampleData_highdim$train
test_dat_highdim <- ExampleData_highdim$test
beta_external_highdim <- ExampleData_highdim$beta_external

model_ridge <- coxkl_ridge(z = train_dat_highdim$z,
                          delta = train_dat_highdim$status,
                          time = train_dat_highdim$time,
                          beta = beta_external_highdim,
                          eta = 1)

plot(
  model_ridge,
  test_z      = test_dat_highdim$z,
  test_time   = test_dat_highdim$time,
  test_delta  = test_dat_highdim$status,
  test_stratum = test_dat_highdim$stratum,
  criteria    = "CIndex"
)
```

---

predict.coxkl      *Predict Linear Predictors from a coxkl Object*

---

### Description

Computes linear predictors for new data based on a fitted coxkl model. If eta is supplied, predictions are returned for those eta values; otherwise predictions are returned for all fitted etas. Linear interpolation is applied if an intermediate eta value is requested.

### Usage

```
## S3 method for class 'coxkl'
predict(object, newz, eta = NULL, ...)
```

### Arguments

object	A fitted model object of class "coxkl".
newz	A numeric matrix or data frame of new covariates (must match the dimension of the training design matrix used to fit the model).
eta	Optional numeric vector of eta value(s) for which to predict. If NULL, predictions for all fitted eta values are returned.
...	Additional arguments.

### Details

The linear predictors are computed as `as.matrix(newz) %*% beta`.

### Value

A numeric matrix of linear predictors with one column per eta (sorted ascending).

### See Also

[coef.coxkl](#)

---

predict.coxkl\_enet      *Predict Linear Predictors from a coxkl\_enet Object*

---

### Description

Computes linear predictors for new data using a fitted coxkl\_enet model. If lambda is supplied, predictions are returned for those lambda values; otherwise predictions are returned for all fitted lambdas. When a requested lambda lies between fitted values, coefficients are linearly interpolated.

**Usage**

```
## S3 method for class 'coxkl_enet'
predict(object, newz, lambda = NULL, ...)
```

**Arguments**

object	A fitted model object of class "coxkl_enet".
newz	A numeric matrix or data frame of new covariates (same columns as in training data).
lambda	Optional numeric value(s) specifying the regularization parameter(s) for which to predict. If NULL, predictions for all fitted lambda values are returned.
...	Additional arguments.

**Details**

The linear predictors are computed as `as.matrix(newz) %*% beta`.

**Value**

A numeric matrix of linear predictors. Each column corresponds to one lambda, sorted in descending order.

**See Also**

[coef.coxkl\\_enet](#)

---

predict.coxkl\_ridge    *Predict Linear Predictors from a coxkl\_ridge Object*

---

**Description**

Computes linear predictors for new data using a fitted coxkl\_ridge model. If lambda is supplied, predictions are returned for those lambda values; otherwise predictions are returned for all fitted lambdas. When a requested lambda lies between fitted values, coefficients are linearly interpolated.

**Usage**

```
## S3 method for class 'coxkl_ridge'
predict(object, newz, lambda = NULL, ...)
```

**Arguments**

object	A fitted model object of class "coxkl_ridge".
newz	A numeric matrix or data frame of new covariates (same columns as in training data).
lambda	Optional numeric value(s) specifying the regularization parameter(s) for which to predict. If NULL, predictions for all fitted lambda values are returned.
...	Additional arguments.

**Details**

The linear predictors are computed as `as.matrix(newz) %*% beta`.

**Value**

A numeric matrix of linear predictors. Each column corresponds to one lambda, sorted in descending order.

**See Also**

[coef.coxkl\\_ridge](#)

---

support	<i>Study to Understand Prognoses Preferences Outcomes and Risks of Treatment</i>
---------	--

---

**Description**

The support dataset tracks five response variables: hospital death, severe functional disability, hospital costs, and time until death and death itself. The patients are followed for up to 5.56 years. See Bhatnagar et al. (2020) for details.

**Usage**

```
data(support)
```

**Format**

A data frame with 9,104 observations and 34 variables after imputation and the removal of response variables like hospital charges, patient ratio of costs to charges and micro-costs following Bhatnagar et al. (2020). Ordinal variables, namely functional disability and income, were also removed. Finally, Surrogate activities of daily living were removed due to sparsity. There were 6 other model scores in the data-set and they were removed; only aps and sps were kept.

**age** stores a double representing age.

**death** death at any time up to NDI (National Death Index) date: 12/31/1994.

**sex** 0=female, 1=male.

**slos** days from study entry to discharge.

**d.time** days of follow-up.

**dzgroup** each level of dzgroup: ARF/MOSF w/Sepsis, COPD, CHF, Cirrhosis, Coma, Colon Cancer, Lung Cancer, MOSF with malignancy.

**dzclass** ARF/MOSF, COPD/CHF/Cirrhosis, Coma and cancer disease classes.

**num.co** the number of comorbidities.

**edu** years of education of patients.

**scoma** the SUPPORT coma score based on Glasgow D3.

**avtisst** average TISS, days 3-25.  
**race** indicates race: White, Black, Asian, Hispanic or other.  
**hday** day in Hospital at Study Admit.  
**diabetes** diabetes (Com27-28, Dx 73).  
**dementia** dementia (Comorbidity 6).  
**ca** cancer state.  
**meanbp** mean arterial blood pressure day 3.  
**wblc** white blood cell count on day 3.  
**hrt** heart rate day 3.  
**resp** respiration rate day 3.  
**temp** temperature, in Celsius, on day 3.  
**pafi** PaO<sub>2</sub>/(0.01\*FiO<sub>2</sub>) day 3.  
**alb** serum albumin day 3.  
**bili** bilirubin day 3.  
**crea** serum creatinine day 3.  
**sod** serum sodium day 3.  
**ph** serum pH (in arteries) day 3.  
**glucose** serum glucose day 3.  
**bun** bun day 3.  
**urine** urine output day 3.  
**adlp** adl patient day 3.  
**adlsc** imputed adl calibrated to surrogate, if a surrogate was used for a follow up.  
**sps** SUPPORT physiology score.  
**aps** apache III physiology score.

## Details

Some of the original data was missing. Before imputation, there were a total of 9,104 individuals and 47 variables. Following Bhatnagar et al. (2020), a few variables were removed. Three response variables were removed: hospital charges, patient ratio of costs to charges and patient micro-costs. Hospital death was also removed as it was directly informative of the event of interest, namely death. Additionally, functional disability and income were removed as they are ordinal covariates. Finally, 8 covariates were removed related to the results of previous findings: SUPPORT day 3 physiology score (sps), APACHE III day 3 physiology score (aps), SUPPORT model 2-month survival estimate, SUPPORT model 6-month survival estimate, Physician's 2-month survival estimate for pt., Physician's 6-month survival estimate for pt., Patient had Do Not Resuscitate (DNR) order, and Day of DNR order (<0 if before study). Of these, sps and aps were added on after imputation, as they were missing only 1 observation. First the imputation is done manually using the normal values for physiological measures recommended by Knaus et al. (1995). Next, a single dataset was imputed using **mice** with default settings. After imputation, the covariate for surrogate activities of daily living was not imputed. This is due to collinearity between the other two covariates for activities of daily living. Therefore, surrogate activities of daily living were removed. See details in the R package (casebase) by Bhatnagar et al. (2020).

## Source

Available at the following website: <https://archive.ics.uci.edu/dataset/880/support2>.

## References

Bhatnagar, S., Turgeon, M., Islam, J., Hanley, J. A., and Saarela, O. (2020) casebase: Fitting Flexible Smooth-in-Time Hazards and Risk Functions via Logistic and Multinomial Regression. *R package version 0.9.0*, <https://CRAN.R-project.org/package=casebase>.

Knaus, W. A., Harrell, F. E., Lynn, J., Goldman, L., Phillips, R. S., Connors, A. F., et al. (1995) The SUPPORT prognostic model: Objective estimates of survival for seriously ill hospitalized adults. *Annals of Internal Medicine*, **122**(3): 191-203.

## Examples

```
if (requireNamespace("survival", quietly = TRUE)) {
  data(support)
  set.seed(123)

  support <- support[support$ca %in% "metastatic", ]
  time <- support$d.time
  death <- support$death
  diabetes <- model.matrix(~ factor(support$diabetes))[, -1]
  # sex: female as the reference group
  sex <- model.matrix(~ support$sex)[, -1]
  # age: continuous variable
  age <- support$age
  age[support$age <= 50] <- "<50"
  age[support$age > 50 & support$age <= 60] <- "50-59"
  age[support$age > 60 & support$age < 70] <- "60-69"
  age[support$age >= 70] <- "70+"
  age <- factor(age, levels = c("60-69", "<50", "50-59", "70+"))
  z_age <- model.matrix(~ age)[, -1]
  z <- data.frame(z_age, sex, diabetes)
  colnames(z) <- c("age_50", "age_50_59", "age_70", "diabetes", "male")
  dat <- data.frame(time, death, z)

  n <- nrow(dat)
  n_ext <- floor(0.87 * n)
  n_int <- floor(0.03 * n)
  n_test <- n - n_ext - n_int
  idx <- sample(seq_len(n))
  idx_ext <- idx[1:n_ext]
  idx_int <- idx[(n_ext + 1):(n_ext + n_int)]
  idx_test <- idx[(n_ext + n_int + 1):n]

  external_data <- dat[idx_ext, ]
  internal_data <- dat[idx_int, ]
  test_data <- dat[idx_test, ]

  ext_cox <- survival::coxph(
```

```

    survival::Surv(time, death) ~ age_50 + age_50_59 + age_70 + diabetes + male,
    data = external_data
  )
  beta_external <- coef(ext_cox)

  result1 <- cv.coxkl(
    z = internal_data[, c("age_50", "age_50_59", "age_70", "diabetes", "male")],
    delta = internal_data$death,
    time = internal_data$time,
    beta = beta_external,
    stratum = NULL,
    etas = generate_eta(method = "exponential", n = 50, max_eta = 50)
  )
  cv.plot(result1)
}

```

---

test\_eval

*Evaluate model performance on test data*


---

### Description

Evaluates model performance on a test dataset using either the log-partial-likelihood loss or the concordance index (C-index).

This function accepts either:

- test\_z and betahat, which will be multiplied to obtain risk scores; or
- test\_RS, a pre-computed numeric vector of risk scores.

### Usage

```

test_eval(
  test_z = NULL,
  test_RS = NULL,
  test_delta,
  test_time,
  test_stratum = NULL,
  betahat = NULL,
  criteria = c("loss", "CIndex")
)

```

### Arguments

test_z	Optional numeric matrix or data frame of covariates for the test dataset. Required if test_RS is not provided.
test_RS	Optional numeric vector of pre-computed risk scores (e.g., linear predictors). If provided, test_z and betahat are ignored.
test_delta	Numeric vector of event indicators (1 = event, 0 = censored).

test_time	Numeric vector of survival times for the test dataset.
test_stratum	Optional vector indicating stratum membership for each test observation. If NULL, all observations are assumed to belong to a single stratum.
betahat	Optional numeric vector of estimated regression coefficients. Required if test_RS is not provided.
criteria	Character string specifying the evaluation criterion; one of: <ul style="list-style-type: none"><li>• "loss": negative twice the log-partial-likelihood.</li><li>• "CIndex": concordance index.</li></ul>

### Details

Prior to evaluation, observations are sorted by (*stratum, time*) to ensure correct risk-set construction. For stratified C-index computation, the provided test\_stratum is used; otherwise all test data are treated as a single stratum.

You may supply either covariates and coefficients (test\_z with betahat) or a precomputed risk score vector (test\_RS). When test\_RS is provided, test\_z and betahat are ignored.

### Value

A numeric value representing either:

- if criteria = "loss": the negative twice log-partial-likelihood on the test data.
- if criteria = "CIndex": the concordance index on the test data.

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