

Regional frequency analysis of the annual flows in Piemonte and Valle d'Aosta

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Abstract

TO BE WRITTEN

Introduction

Many practical hydrological problems require reliable models for estimation of mean annual runoff in a region. Runoff cannot be interpolated like purely distributed variables, as precipitation or temperature, because runoff in a cross section is representative of the whole contributing basin. Therefore, usual spatial interpolation methods cannot be used for estimation in ungauged basins. As regards the statistical approach, one of the firsts and more popular methods in regional frequency analysis is the “index-flood” technique (Dalrymple, 1960). Many Regional Flood estimation projects (see e.g. Rossi and Villani, 1995; Robson and Reed, 1999) are based on Dalrymple’s methodology, but also flow duration curves can be referred to the index flow method (Claps and Fiorentino, 1997; Castellarin et al., 2004a,b).

In this work we are interested in the annual flow, that is the amount of water crossing a river section in one year. If compared with hydrological extremes, applications of regional analysis to average variables, like the annual flow, are much less frequent in literature. Vogel and Wilson (1996) present some applications related to the US, while in Italy some previous works can be traced back to Ferraresi et al. (1988), Claps and Mancino (2002) and Brath et al. (2004). The purpose of the Regional frequency analysis of the annual flow is the estimation of its probability distribution in basins with few or no data.

The fundamental hypothesis of Dalrymple’s method is that the distribution of a variable in different sites belonging to a “homogeneous region” is identical, with the exception of the scale parameter, the index-flow. In this document we show how the `nsRFA` package can be used to:

1. regionalize the index-flow;
2. regionalize the growth curve, i.e. the rescaled distribution function.

The methodology has been applied to Piemonte and Valle d'Aosta, two contiguous regions in the North-West of Italy. This territory is characterized by a marked heterogeneity. In this relatively small region, very different orographic and climatic conditions coexist: in few hundreds kilometres the climate changes from the appenninic-mediterranean one in the south-eastern hills to the alpine-continental one in the mountainous Valle d'Aosta, passing from all the intermediate conditions. For this reason, a regional frequency analysis in this territory is both complex and interesting.

The following results are documented in Viglione et al. (2006) and Viglione (2007).

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Data

In `nsRFA` data referred to 47 basins in Piemonte and Valle d'Aosta are in:

```
> data(hydroSIMN)
```

To have some information on these data

```
> ls()
> help(hydroSIMN)
```

The object used in this work are `annualflows`, a data.frame containing the annual flows of 47 hydrometric stations in Piemonte and Valle d'Aosta, measured by the SIMN (Servizio Idrografico e Mareografico Nazionale), and `parameters`, a data.frame containing morphometric and climatic descriptors that have been derived for all these river basins.

Regionalization of the index-flow

The “index-flow” parameter can be either the sample mean (e.g. Hosking and Wallis, 1997) or the sample median (e.g. Robson and Reed, 1999). Viglione et al. (2007) show that, for variables characterized by low skewness coefficients, the estimation of the mean is less biased than that of the median. For this reason in this work the sample mean is used as the index-flow. Due to its simplicity, the most frequently used method to estimate the index-flow is the multiregressive approach (see e.g. Kottek and Rosso, 1997), that relates the index-flow to catchment characteristics, such as climatic indices, geologic and morphologic parameters, land cover type, etc., through linear (used here) or non-linear equations.

The choice of the best linear regressions between the mean annual flow and the catchment attributes is performed using the function `bestlm()`. Different types of linear models are investigated. The candidate dependent variable is selected between 4 possibilities:

```
> Dm <- parameters[, "Dm"]
> logDm <- log(Dm)
> sqrtDm <- sqrt(Dm)
> sqrt3Dm <- Dm^(1/3)
```

The candidate regressors are:

```
> attributes <- parameters[, -c(1, 2)]
> logattributes <- log(attributes[, -c(7:9)])
> mixedattributes <- cbind(attributes, logattributes[, 1])
> names(mixedattributes) <- c(names(attributes), "lnAm")
```

The best regressions with the non transformed mean annual flows are:

```
> nontrasfregr <- bestlm(Dm, mixedattributes, kmax = 3, nbest = 4)
> nontrasfregr
```

| | model | R2adj |
|---|-------------------|-----------|
| 1 | Am + S2000 + IT | 0.8851092 |
| 2 | S2000 + IT + lnAm | 0.8845462 |
| 3 | Am + S2000 + IB | 0.8793968 |
| 4 | Am + S2000 + lnAm | 0.8789050 |
| 5 | S2000 + lnAm | 0.8773823 |
| 6 | Am + S2000 | 0.8761521 |
| 7 | Hm + lnAm | 0.8648352 |
| 8 | Am + Hm | 0.8588129 |
| 9 | IT | 0.8467383 |

```

10          IB 0.6094562
11      lnAm 0.5440745
12          Am 0.5323001

```

Other diagnostics of these regressions can be obtained using the functions in **REGRDIAGNOSTICS**. Here we calculate the Root Mean Squared Error (RMSE), and the Root Mean Squared Error of the cross validation (RMSEjk)

```

> nregr <- dim(nontrasfregr$subselect)[1]
> diagn <- data.frame(matrix(NA, nrow = nregr, ncol = 2))
> names(diagn) <- c("RMSE", "RMSEjk")
> for (i in 1:nregr) {
+   f <- paste("Dm ~", paste(colnames(nontrasfregr$subselect)[nontrasfregr$subselect[i],
+     ]], collapse = " + "))
+   regr <- lm(f, mixedattributes)
+   diagn[i, 1] <- RMSE.lm(regr)
+   diagn[i, 2] <- RMSEjk.lm(regr)
+ }
> diagn

      RMSE    RMSEjk
1 104.0258 113.4305
2 104.2804 114.1658
3 106.5805 116.2866
4 106.7976 116.3828
5 108.7094 116.6226
6 109.2534 116.9209
7 114.1359 122.2157
8 116.6509 124.9485
9 122.9100 127.3007
10 196.2029 203.4998
11 211.9910 219.7052
12 214.7109 222.3708

```

The best multiplicative regressions (linearized with the logarithms) are:

```

> multregr <- bestlm(logDm, logattributes, kmax = 3, nbest = 4)
> multregr

```

| | model | R2adj |
|----|----------------|-----------|
| 1 | Hm + LLDP + IB | 0.8468383 |
| 2 | S + Hm + IB | 0.8467204 |
| 3 | Am + Hm + LLDP | 0.8458270 |
| 4 | Am + S + Hm | 0.8447653 |
| 5 | Am + Hm | 0.8330193 |
| 6 | Hm + IB | 0.8245532 |
| 7 | IT + IB | 0.8069278 |
| 8 | Am + IT | 0.8015272 |
| 9 | IT | 0.7787654 |
| 10 | IB | 0.5571305 |
| 11 | Am | 0.5172267 |
| 12 | Pm | 0.3590828 |

whose RMSE and RMSEjk are:

```
> nregr <- dim(multregr$subselect)[1]
> diagn <- data.frame(matrix(NA, nrow = nregr, ncol = 2))
> names(diagn) <- c("RMSE", "RMSEjk")
> for (i in 1:nregr) {
+   f <- paste("logDm ~", paste(colnames(multregr$subselect)[multregr$subselect[i,
+     ]], collapse = " + "))
+   regr <- lm(f, logattributes)
+   fitt <- regr$fitted.values
+   crossval <- jackknife1.lm(regr)
+   diagn[i, 1] <- RMSE(Dm, exp(fitt))
+   diagn[i, 2] <- RMSE(Dm, exp(crossval))
+ }
> diagn
```

| | RMSE | RMSEjk |
|----|----------|----------|
| 1 | 129.3508 | 141.2024 |
| 2 | 128.9386 | 141.0293 |
| 3 | 127.9561 | 139.3443 |
| 4 | 127.5461 | 139.2905 |
| 5 | 130.8245 | 139.3654 |
| 6 | 135.7904 | 145.6343 |
| 7 | 140.2605 | 148.5320 |
| 8 | 141.6007 | 150.6099 |
| 9 | 149.1973 | 156.0839 |
| 10 | 214.6567 | 224.6109 |
| 11 | 217.2887 | 225.2206 |
| 12 | 266.0818 | 275.1440 |

The best regressions with the transformed mean annual flows are:

```
> trasfregr_log <- bestlm(logDm, mixedattributes, kmax = 3, nbest = 4)
> trasfregr_log
```

| | model | R2adj |
|----|-------------------|-----------|
| 1 | Hm + NORD + IB | 0.9001788 |
| 2 | Hm + NORD + lnAm | 0.8922544 |
| 3 | Hm + EST + lnAm | 0.8893036 |
| 4 | Hm + S2000 + lnAm | 0.8885439 |
| 5 | S2000 + lnAm | 0.8838486 |
| 6 | Hm + lnAm | 0.8828080 |
| 7 | Hm + IB | 0.8823572 |
| 8 | S2000 + IB | 0.8723769 |
| 9 | IT | 0.8382124 |
| 10 | IB | 0.5944382 |
| 11 | lnAm | 0.5172267 |
| 12 | Am | 0.4877632 |

```
> nregr <- dim(trasfregr_log$subselect)[1]
> diagn <- data.frame(matrix(NA, nrow = nregr, ncol = 2))
> names(diagn) <- c("RMSE", "RMSEjk")
```

```

> for (i in 1:nregr) {
+   f <- paste("logDm ~", paste(colnames(trasfregr_log$subselect)[trasfregr_log$subselect[i],
+                                ], collapse = " + "))
+   regr <- lm(f, mixedattributes)
+   fitt <- regr$fitted.values
+   crossval <- jackknife1.lm(regr)
+   diagn[i, 1] <- RMSE(Dm, exp(fitt))
+   diagn[i, 2] <- RMSE(Dm, exp(crossval))
+ }
> diagn

      RMSE    RMSEjk
1 101.7852 110.5497
2 103.2933 112.9716
3 101.7492 111.8125
4 103.0333 112.0052
5 106.2369 113.4683
6 108.6799 116.1945
7 111.3227 118.5316
8 117.3381 125.3689
9 136.1306 144.1668
10 202.3507 210.0492
11 217.2887 225.2206
12 226.6542 236.5474

> trasfregr_sqrt <- bestlm(sqrtDm, mixedattributes, kmax = 3, nbest = 4)
> trasfregr_sqrt

      model      R2adj
1  S2000 + IT + lnAm 0.8936962
2  Hm + S2000 + lnAm 0.8893312
3  Hm + NORD + IB 0.8893250
4  S2000 + lnAm 0.8877719
5  S2000 + NORD + lnAm 0.8876825
6  Hm + lnAm 0.8801759
7  Hm + IB 0.8747952
8  Am + S2000 0.8738923
9  IT 0.8490673
10  IB 0.6065118
11  lnAm 0.5348015
12  Am 0.5137850

> nregr <- dim(trasfregr_sqrt$subselect)[1]
> diagn <- data.frame(matrix(NA, nrow = nregr, ncol = 2))
> names(diagn) <- c("RMSE", "RMSEjk")
> for (i in 1:nregr) {
+   f <- paste("sqrtDm ~", paste(colnames(trasfregr_sqrt$subselect)[trasfregr_sqrt$subselect[i],
+                                ], collapse = " + "))
+   regr <- lm(f, mixedattributes)
+   fitt <- regr$fitted.values
+   crossval <- jackknife1.lm(regr)

```

```

+      diagn[i, 1] <- RMSE(Dm, fitt^2)
+      diagn[i, 2] <- RMSE(Dm, crossval^2)
+ }
> diagn

      RMSE     RMSEjk
1 100.5960 109.7789
2 103.8121 113.0390
3 105.1730 114.7534
4 105.9916 113.4752
5 105.2430 115.6963
6 109.2252 116.6429
7 112.8295 120.2861
8 110.4650 118.5180
9 124.9163 129.9945
10 197.6366 204.9570
11 213.2123 220.8817
12 218.5031 226.6602

> trasfregr_sqrt3 <- bestlm(sqrt3Dm, mixedattributes, kmax = 3,
+      nbest = 4)
> trasfregr_sqrt3

      model      R2adj
1      Hm + NORD + IB 0.8944924
2  S2000 + IT + lnAm 0.8936365
3  Hm + S2000 + lnAm 0.8906543
4  Hm + NORD + lnAm 0.8906427
5  S2000 + lnAm 0.8881545
6      Hm + lnAm 0.8825746
7      Hm + IB 0.8788041
8  S2000 + IB 0.8725909
9      IT 0.8469916
10      IB 0.6035465
11      lnAm 0.5298848
12      Am 0.5059347

> nregr <- dim(trasfregr_sqrt3$subselect)[1]
> diagn <- data.frame(matrix(NA, nrow = nregr, ncol = 2))
> names(diagn) <- c("RMSE", "RMSEjk")
> for (i in 1:nregr) {
+   f <- paste("sqrt3Dm ~", paste(colnames(trasfregr_sqrt3$subselect)[trasfregr_sqrt3$subsel
+     ]], collapse = " + "))
+   regr <- lm(f, mixedattributes)
+   fitt <- regr$fitted.values
+   crossval <- jackknife1.lm(regr)
+   diagn[i, 1] <- RMSE(Dm, fitt^3)
+   diagn[i, 2] <- RMSE(Dm, crossval^3)
+ }
> diagn

```

| | RMSE | RMSEjk |
|----|----------|----------|
| 1 | 103.6245 | 112.8929 |
| 2 | 100.1596 | 109.2270 |
| 3 | 103.1318 | 112.2067 |
| 4 | 103.9504 | 113.9002 |
| 5 | 105.7137 | 113.0890 |
| 6 | 108.4943 | 115.8495 |
| 7 | 111.8762 | 119.2041 |
| 8 | 116.0246 | 124.1258 |
| 9 | 127.2240 | 132.8930 |
| 10 | 198.7465 | 206.1318 |
| 11 | 214.1709 | 221.8712 |
| 12 | 220.5780 | 229.0946 |

The choice of the best regression is based on the RMSE of the cross-validation (function `RMSEjk.lm` or `jackknife1.lm` plus `RMSE`). So the best regression is:

```
> bestregr <- lm(sqrt3Dm ~ S2000 + IT + lnAm, mixedattributes)
> bestregr
```

Call:

```
lm(formula = sqrt3Dm ~ S2000 + IT + lnAm, data = mixedattributes)
```

Coefficients:

| (Intercept) | S2000 | IT | lnAm |
|-------------|---------|---------|---------|
| -14.86525 | 0.01601 | 0.71038 | 3.32829 |

```
> summary(bestregr)
```

Call:

```
lm(formula = sqrt3Dm ~ S2000 + IT + lnAm, data = mixedattributes)
```

Residuals:

| Min | 1Q | Median | 3Q | Max |
|----------|----------|----------|---------|---------|
| -0.84607 | -0.16793 | -0.01297 | 0.14732 | 0.80262 |

Coefficients:

| | Estimate | Std. Error | t value | Pr(> t) |
|-------------|-----------|------------|---------|--------------|
| (Intercept) | -14.86525 | 5.72525 | -2.596 | 0.012840 * |
| S2000 | 0.01601 | 0.00344 | 4.655 | 3.11e-05 *** |
| IT | 0.71038 | 0.39298 | 1.808 | 0.077653 . |
| lnAm | 3.32829 | 0.83259 | 3.998 | 0.000247 *** |

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.3457 on 43 degrees of freedom

Multiple R-squared: 0.9006, Adjusted R-squared: 0.8936

F-statistic: 129.8 on 3 and 43 DF, p-value: < 2.2e-16

that we check with the following tests: the Variance inflation factors (if $VIF > 5$ there is a problem of multicollinearity) and correlation between the regressors:

```

> vif.lm(bestregr)

      S2000       IT       lnAm
4.692027 11.211280 13.021852

> cor(bestregr$model[-1])

      S2000       IT       lnAm
S2000  1.0000000 0.1500960 -0.398042
IT      0.1500960 1.0000000  0.804859
lnAm   -0.3980420 0.8048590  1.000000

```

the Student t test of significance of the coefficients (probability $Pr(>|t|)$ of the significance test, the smallest-the best):

```

> prt.lm(bestregr)

      S2000       IT       lnAm
3.105746e-05 7.765271e-02 2.469395e-04

```

So there is a correlation problem between IT and lnAm, that can cause collinearity, and that causes the non-significance of the coefficient of IT in the model.

Therefore we choose:

```

> bestregr <- lm(logDm ~ Hm + NORD + IB, mixedattributes)
> bestregr

```

Call:
`lm(formula = logDm ~ Hm + NORD + IB, data = mixedattributes)`

Coefficients:

| (Intercept) | Hm | NORD | IB |
|-------------|-----------|-----------|------------|
| 7.8577186 | 0.0002910 | 0.0722161 | -1.6956356 |

```
> summary(bestregr)
```

Call:
`lm(formula = logDm ~ Hm + NORD + IB, data = mixedattributes)`

Residuals:

| Min | 1Q | Median | 3Q | Max |
|-----------|-----------|----------|----------|----------|
| -0.273533 | -0.059161 | 0.007672 | 0.041782 | 0.244361 |

Coefficients:

| | Estimate | Std. Error | t value | Pr(> t) |
|-------------|------------|------------|---------|--------------|
| (Intercept) | 7.858e+00 | 7.849e-02 | 100.117 | < 2e-16 *** |
| Hm | 2.910e-04 | 2.465e-05 | 11.807 | 4.42e-15 *** |
| NORD | 7.222e-02 | 2.427e-02 | 2.976 | 0.00478 ** |
| IB | -1.696e+00 | 9.503e-02 | -17.843 | < 2e-16 *** |

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.1018 on 43 degrees of freedom
Multiple R-squared: 0.9067, Adjusted R-squared: 0.9002
F-statistic: 139.3 on 3 and 43 DF, p-value: < 2.2e-16

```
> prt.lm(bestregr)

      Hm          NORD          IB
4.420213e-15 4.780480e-03 1.707744e-21
```

```
> vif.lm(bestregr)

      Hm          NORD          IB
1.148364 1.330344 1.336530
```

```
> cor(bestregr$model[-1])
```

```
      Hm          NORD          IB
Hm    1.0000000 -0.1887886 0.2002600
NORD -0.1887886 1.0000000 0.4140169
IB    0.2002600 0.4140169 1.0000000
```

We also check the normality of the residuals (using a goodness-of-fit test):

```
> p_norm <- A2_GOFlaio(bestregr$residuals, dist = "NORM")
> p_norm
```

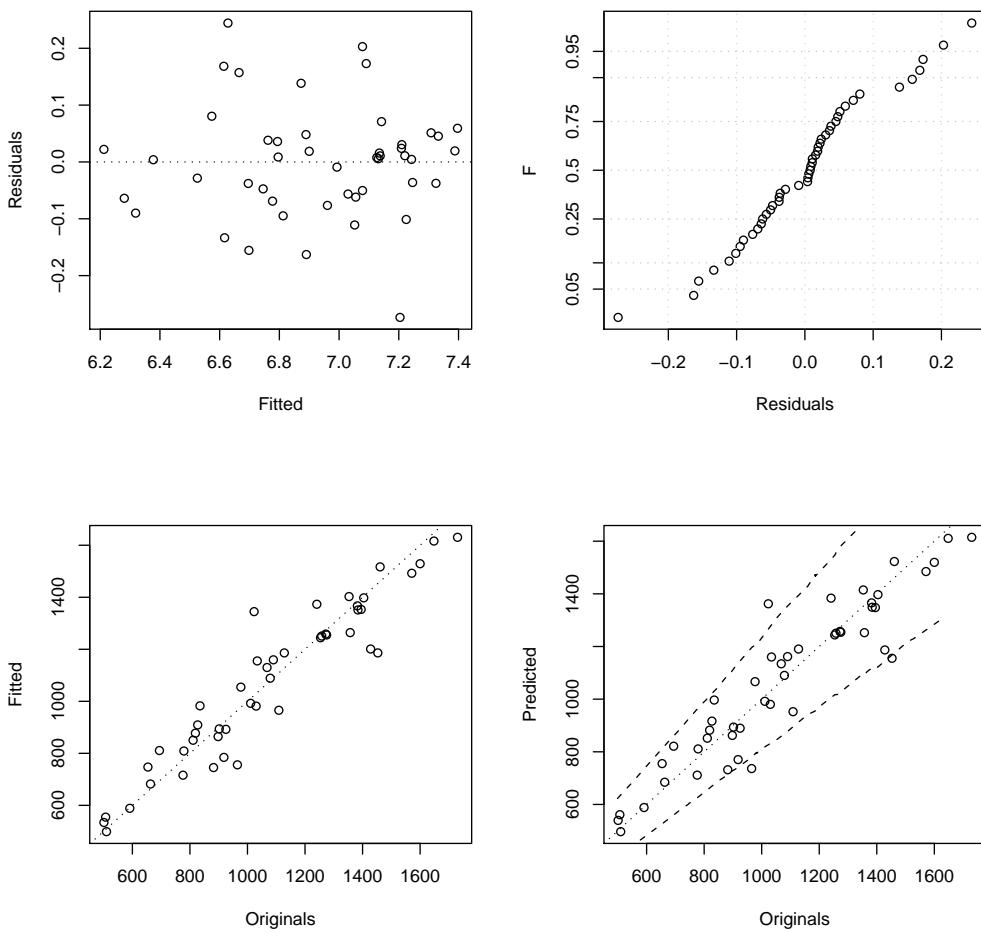
```
      A2      p(A2)
0.5963062 0.8765066
```

and calculate the root mean square errors:

```
> rmse <- RMSE(Dm, exp(bestregr$fitted.values))
> predicted <- jackknife1.lm(bestregr)
> rmse_jk <- RMSE(Dm, exp(predicted))
```

Some diagnostics can be plotted as:

```
> op <- par(mfrow = c(2, 2))
> plot(bestregr$fitted.values, bestregr$residuals, xlab = "Fitted",
+       ylab = "Residuals")
> abline(0, 0, lty = 3)
> normplot(bestregr$residuals, xlab = "Residuals")
> plot(parameters[, c("Dm")], exp(bestregr$fitted.values), xlab = "Originals",
+       ylab = "Fitted")
> abline(0, 1, lty = 3)
> intervals <- predinterval.lm(bestregr)
> intervals <- intervals[order(intervals[, 1]), ]
> plot(parameters[, c("Dm")], exp(predicted), xlab = "Originals",
+       ylab = "Predicted")
> abline(0, 1, lty = 3)
> lines(exp(intervals[, c(1, 2)]), lty = 2)
> lines(exp(intervals[, c(1, 3)]), lty = 2)
> par(op)
```



Diagnostic plots of the best regression model. Counterclockwise from upper left: residuals as a function of the estimated values; originals against the fitted values; result of cross-validation and normal plot of residuals.

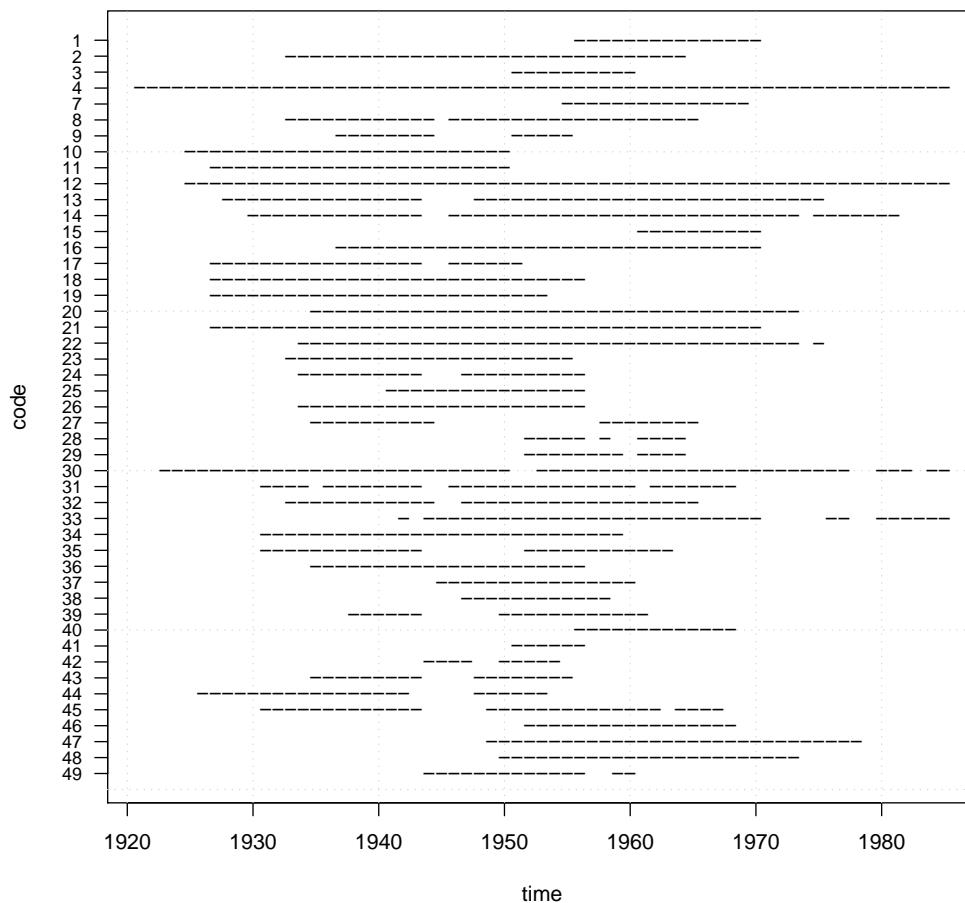
Regionalization of the growth-curve

TO BE COMPLETED...

```
> D <- annualflows["dato"][, ]
> y <- annualflows["anno"][, ]
> cod <- annualflows["cod"][, ]
```

Plot consistency of data series:

```
> consistencyplot(y, cod)
```



Data consistency.

Choice of sites with more than 15 records:

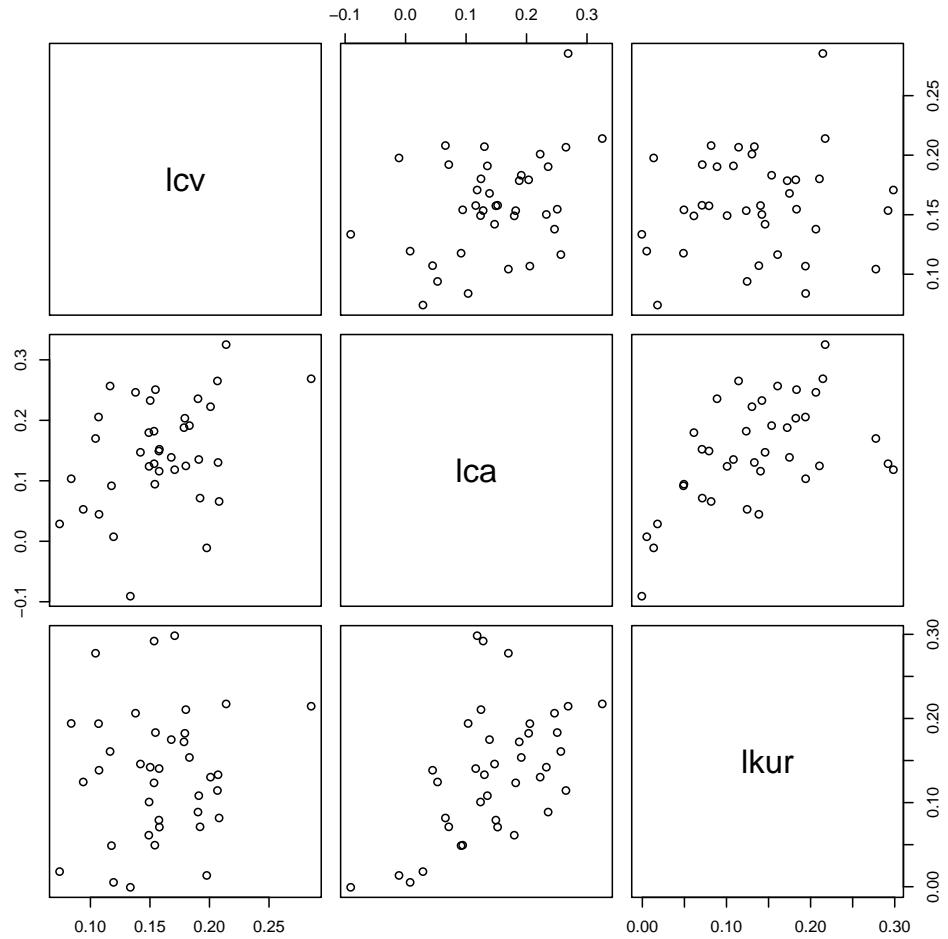
```
> ni <- tapply(D, cod, length)
> annualflows15 <- annualflows[unsplit(ni, cod) >= 15, ]
> parameters15 <- parameters[ni >= 15, ]
> D15 <- annualflows15["dato"][, ]
> cod15 <- annualflows15["cod"][, ]
```

L-moments of the series:

```
> LM15 <- data.frame(t(sapply(split(D15, cod15), Lmoments)))
```

L-moment ratios plot:

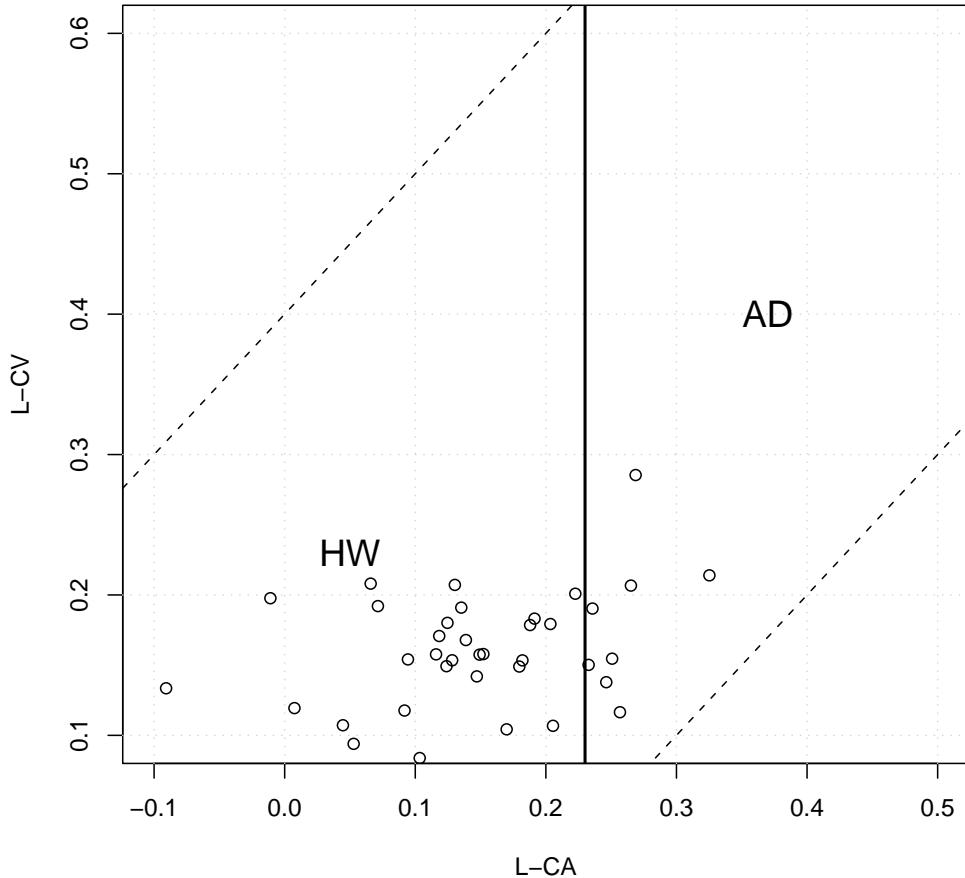
```
> plot(LM15[3:5])
```



L-moment ratios plot.

Which homogeneity test do I use:

```
> Lspace.HWvsAD()  
> points(LM15[, 4:3])
```



L-moment ratios plot.

Homogeneity test on the entire region:

```
> D15adim <- D15/unsplit(tapply(D15, cod15, mean), cod15)
> HWs <- HW.tests(D15adim, cod15)[1]
> HWs
```

```
H1
7.776372
```

Choice of the classification variables through multi-regression approach:

```
> bestlm(LM15[, "lcv"], parameters15[, 3:16], kmax = 3)
```

| | model | R2adj |
|---|-------------------|-----------|
| 1 | S2000 + Rc + IB | 0.6780821 |
| 2 | Am + S2000 + Rc | 0.6767409 |
| 3 | Am + LLDP + S2000 | 0.6762082 |
| 4 | S2000 + Ybar | 0.6595223 |
| 5 | Hm + Ybar | 0.6435752 |
| 6 | Am + S2000 | 0.6306392 |
| 7 | S2000 | 0.5394290 |
| 8 | Hm | 0.5321995 |
| 9 | Ybar | 0.3584742 |

or reasoning with distance matrices:

```
> bestlm(as.numeric(AD.dist(D15, cod15)), data.frame(apply(parameters15[,  
+      3:16], 2, dist)), kmax = 3)

      model      R2adj  
1 Am + S2000 + Ybar 0.16116558  
2 S2000 + EST + Ybar 0.15822746  
3 Pm + S2000 + Ybar 0.15718520  
4      S2000 + Ybar 0.14887002  
5      Hm + Ybar 0.14154585  
6      S2000 + EST 0.11684503  
7      S2000 0.10917391  
8      Hm 0.09862238  
9      Ybar 0.05313668
```

We choose Hm and Ybar as classification variables. Mantel test:

```
> Y <- AD.dist(D15, cod15)  
> X <- data.frame(apply(parameters15[, c("Hm", "Ybar")], 2, dist))  
> datamantel <- cbind(as.numeric(Y), X)  
> regrmantel <- lm(Y ~ Hm + Ybar, datamantel)  
> mantel.lm(regrmantel, Nperm = 100)
```

```
P.Hm P.Ybar  
1      1
```

Cluster formation:

```
> param <- parameters15[c("Hm", "Ybar")]  
> n <- dim(param)[1]  
> k <- dim(param)[2]  
> param.norm <- (param - matrix(mean(param), nrow = n, ncol = k,  
+   byrow = TRUE))/matrix(sd(param), nrow = n, ncol = k, byrow = TRUE)  
  
> nclusters = 1  
> while (max(HWs) > 2.1) {  
+   nclusters <- nclusters + 1  
+   clusters <- traceWminim(param.norm, nclusters)  
+   indclusters <- unsplit(clusters, cod15)  
+   HWs <- rep(NA, nclusters)  
+   for (i in unique(clusters)) {  
+     HWs[i] <- HW.tests(D15adim[indclusters == i], cod15[indclusters ==  
+       i])[1]  
+   }  
+   print(HWs)  
+ }
```

```
[1] 4.426389 1.617630  
[1] 4.7201696 -0.1214777 0.4974403  
[1] 1.7858483 1.9609622 -0.2935966 0.4070506
```

So the number of regions is 4.

Regional L-moments: My regional L-moment ratios are not calculated as in Hosking and Wallis, but using the pooled growth-curves.

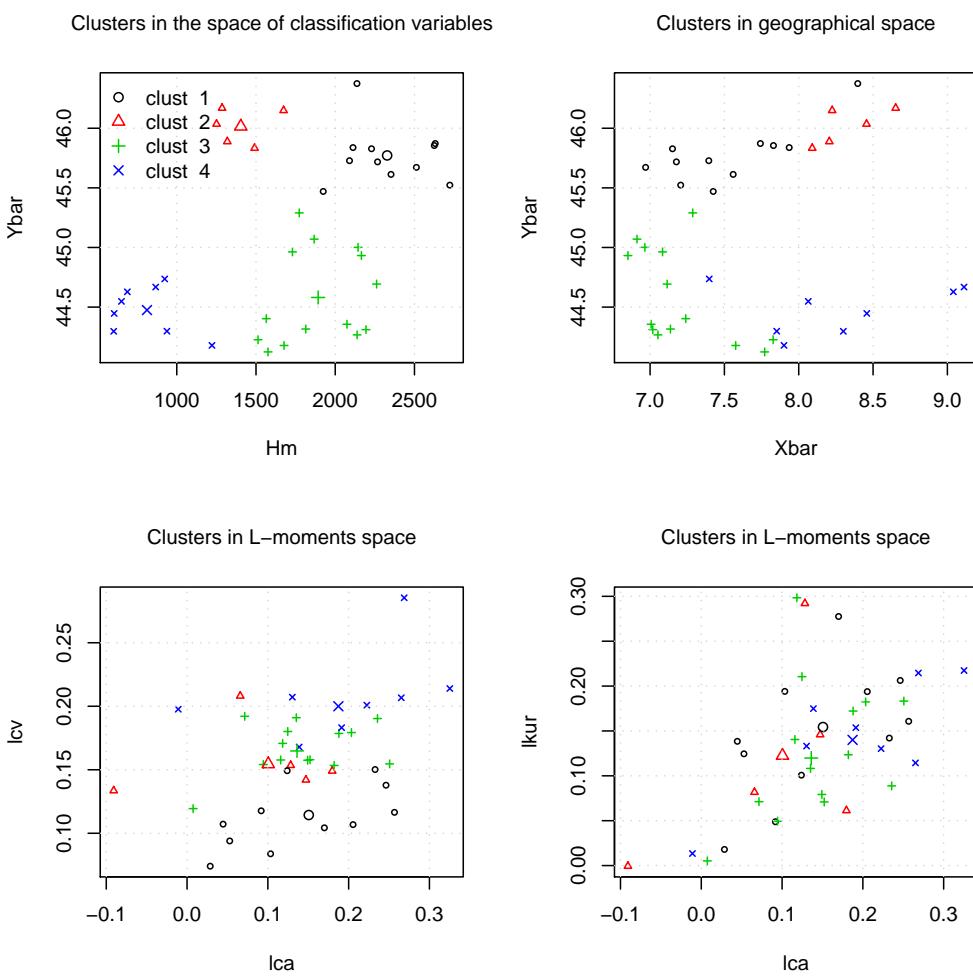
```
> regLM15 <- t(sapply(split(D15adim, indclusters), Lmoments))[,  
+      3:5]  
> regLM15  
  
      lcv      lca      lkur  
1 0.1143714 0.1506529 0.1543626  
2 0.1544286 0.1005249 0.1224144  
3 0.1648291 0.1361256 0.1197992  
4 0.1999977 0.1872976 0.1400214
```

If I calculate them with the method of Hosking and Wallis:

```
> for (i in 1:nclusters) {  
+   print(regionallmoments(D15adim[indclusters == i], cod15[indclusters ==  
+     i])[3:5])  
+ }  
  
      lcvR      lcaR      lkurR  
0.1156601 0.1550459 0.1489688  
      lcvR      lcaR      lkurR  
0.1557402 0.1021739 0.1096460  
      lcvR      lcaR      lkurR  
0.1680399 0.1406213 0.1180470  
      lcvR      lcaR      lkurR  
0.2032194 0.1938267 0.1438153
```

Plot of clusters:

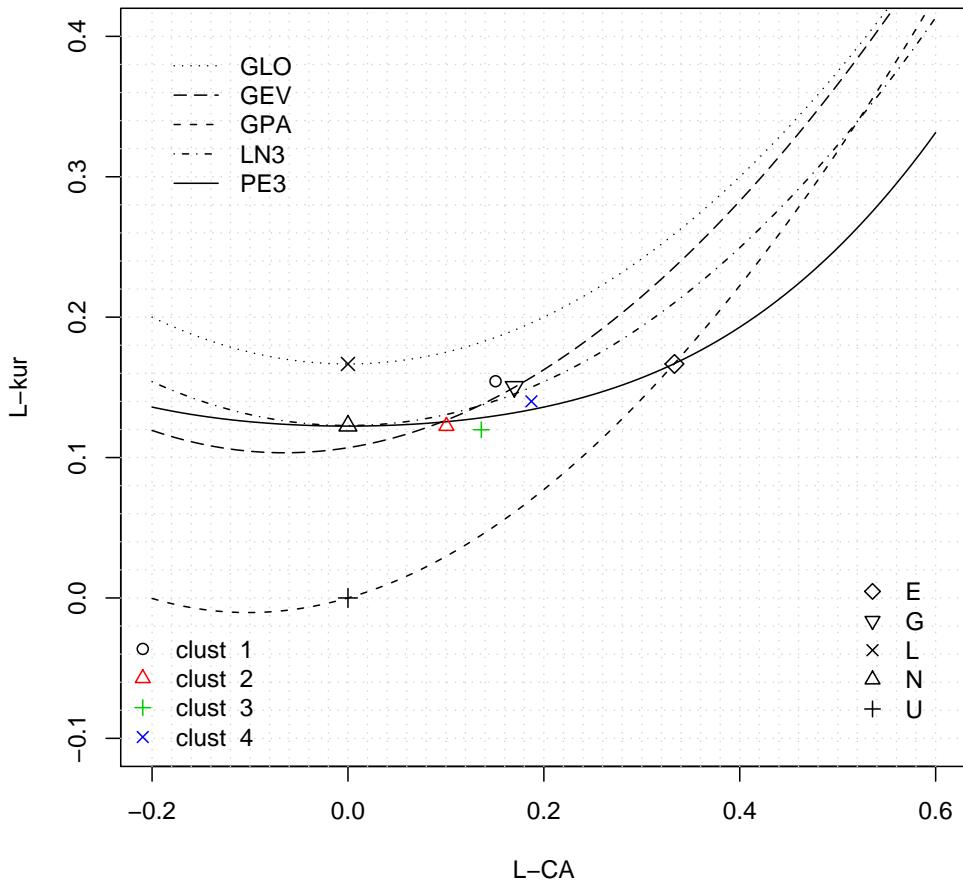
```
> op <- par(mfrow = c(2, 2))  
> plot(parameters15[c("Hm", "Ybar")], col = clusters, pch = clusters,  
+       cex = 0.6, main = "Clusters in the space of classification variables",  
+       cex.main = 1, font.main = 1)  
> grid()  
> points(tapply(parameters15["Hm"][, ], clusters, mean), tapply(parameters15["Ybar"][,  
+     ], clusters, mean), col = c(1:nclusters), pch = c(1:nclusters))  
> legend("topleft", paste("clust ", c(1:nclusters)), col = c(1:nclusters),  
+       pch = c(1:nclusters), bty = "n")  
> plot(parameters15[c("Xbar", "Ybar")], col = clusters, pch = clusters,  
+       cex = 0.6, main = "Clusters in geographical space", cex.main = 1,  
+       font.main = 1)  
> grid()  
> plot(LM15[, 4:3], pch = clusters, col = clusters, cex = 0.6,  
+       main = "Clusters in L-moments space", cex.main = 1, font.main = 1)  
> points(regLM15[, 2:1], col = c(1:nclusters), pch = c(1:nclusters))  
> grid()  
> plot(LM15[, 4:5], pch = clusters, col = clusters, cex = 0.6,  
+       main = "Clusters in L-moments space", cex.main = 1, font.main = 1)  
> points(regLM15[, 2:3], col = c(1:nclusters), pch = c(1:nclusters))  
> grid()  
> par(op)
```



Clusters.

Model selection (L-moments ratio diagram):

```
> Lmoment.ratio.diagram()
> points(regLM15[, 2:3], col = c(1:nclusters), pch = c(1:nclusters))
> legend("bottomleft", paste("clust ", c(1:nclusters)), col = c(1:nclusters),
+       pch = c(1:nclusters), bty = "n")
```



L-moments ratio diagram.

The points are around the Pearson type III distribution. If we apply the Anderson-Darling goodness-of-fit test, we obtain:

```
> for (i in 1:nclusters) {
+   GOFA2_P3 <- A2_GOFlaio(D15adim[indclusters == i], dist = "P3")
+   cat(paste("\np(A2) for Cluster ", i, ":\n"), sep = ""))
+   print(GOFA2_P3)
+ }

p(A2) for Cluster 1:
      A2      p(A2)
0.3152134 0.5608133

p(A2) for Cluster 2:
      A2      p(A2)
0.2531199 0.3859892

p(A2) for Cluster 3:
      A2      p(A2)
0.5713187 0.9220134
```

```
p(A2) for Cluster 4:
      A2      p(A2)
0.6638851 0.9532153
```

For the 4-th cluster, the goodness of fit test is not passed with a 5% significance level.

Parameters of the Pearson type III distributions using the method of L-moments:

```
> paramgamma = NULL
> for (i in 1:nclusters) {
+   paramgamma[[i]] <- par.gamma(1, regLM15[i, 1], regLM15[i,
+     2])
+   cat(paste("\nCluster", i, ":\n"))
+   print(format(paramgamma[[i]][1:3]))
+ }
```

Cluster 1 :

| xi | beta | alfa |
|-------------|--------------|------------|
| "0.5457898" | "0.09534035" | "4.764092" |

Cluster 2 :

| xi | beta | alfa |
|--------------|--------------|------------|
| "0.09842929" | "0.08508402" | "10.59624" |

Cluster 3 :

| xi | beta | alfa |
|-------------|-------------|------------|
| "0.2800572" | "0.1237543" | "5.817518" |

Cluster 4 :

| xi | beta | alfa |
|-------------|-------------|------------|
| "0.3496308" | "0.2093341" | "3.106847" |

Expressed in the other way:

```
> for (i in 1:nclusters) {
+   cat(paste("\nCluster", i, ":\n"))
+   print(format(par2mom.gamma(paramgamma[[i]]$xi, paramgamma[[i]]$beta,
+     paramgamma[[i]]$alfa)))
+ }
```

Cluster 1 :

| mu | sigma | gamm |
|------------|--------------|------------|
| "4.816127" | "0.07043515" | "2.707181" |

Cluster 2 :

| mu | sigma | gamm |
|------------|--------------|------------|
| "10.60462" | "0.02669378" | "6.374819" |

Cluster 3 :

| mu | sigma | gamm |
|------------|--------------|------------|
| "5.852176" | "0.06549131" | "3.779259" |

Cluster 4 :

```

mu           sigma        gamm
"3.180037" "0.1237784" "3.382401"

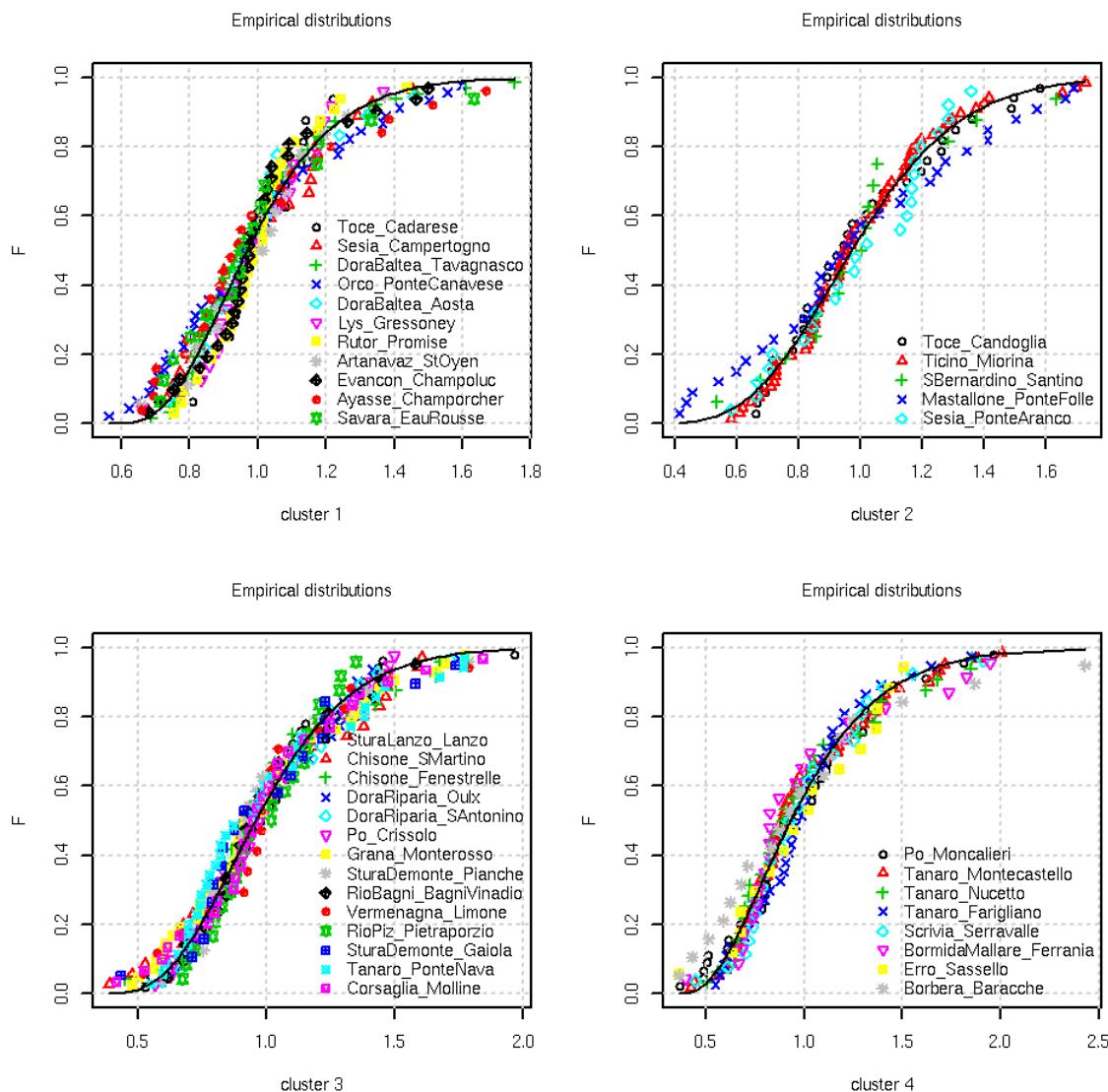
```

Regional growth-curves:

```

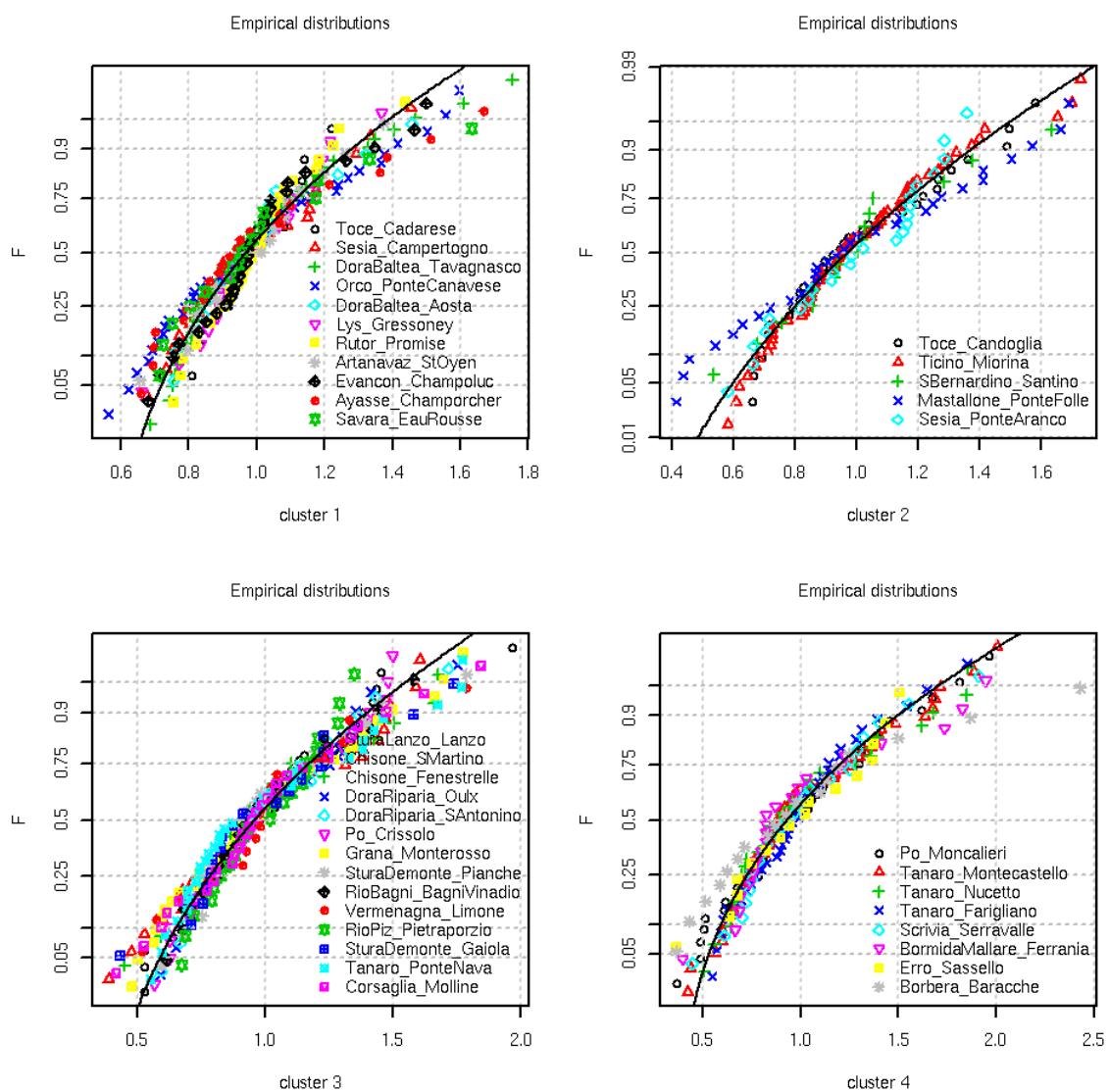
> op <- par(mfrow = c(2, 2))
> for (i in 1:nclusters) {
+   FF <- F.gamma(D15adim[indclusters == i], paramgamma[[i]]$xi,
+                 paramgamma[[i]]$beta, paramgamma[[i]]$alfa)
+   regionalplotpos(D15adim[indclusters == i], cod15[indclusters ==
+               i], xlab = paste("cluster", i), main = "Empirical distributions",
+               cex.main = 1, font.main = 1)
+   lines(sort(D15adim[indclusters == i]), sort(FF))
+   nomi <- names(clusters)[clusters == i]
+   legend("bottomright", legend = nomi, pch = c(1:length(nomi)),
+         col = c(1:length(nomi)), bty = "n", cex = 0.9)
+ }
> par(op)

```



Regional growth-curves:

```
> op <- par(mfrow = c(2, 2))
> for (i in 1:nclusters) {
+   Fs <- seq(0.001, 0.999, by = 0.001)
+   regionalnormplot(D15adim[indclusters == i], cod15[indclusters ==
+     i], xlab = paste("cluster", i), main = "Empirical distributions",
+     cex.main = 1, font.main = 1)
+   normpoints(invF.gamma(Fs, paramgamma[[i]]$xi, paramgamma[[i]]$beta,
+     paramgamma[[i]]$alpha), type = "l")
+   nomi <- names(clusters)[clusters == i]
+   legend("bottomright", legend = nomi, pch = c(1:length(nomi)),
+     col = c(1:length(nomi)), bty = "n", cex = 0.9)
+ }
> par(op)
```

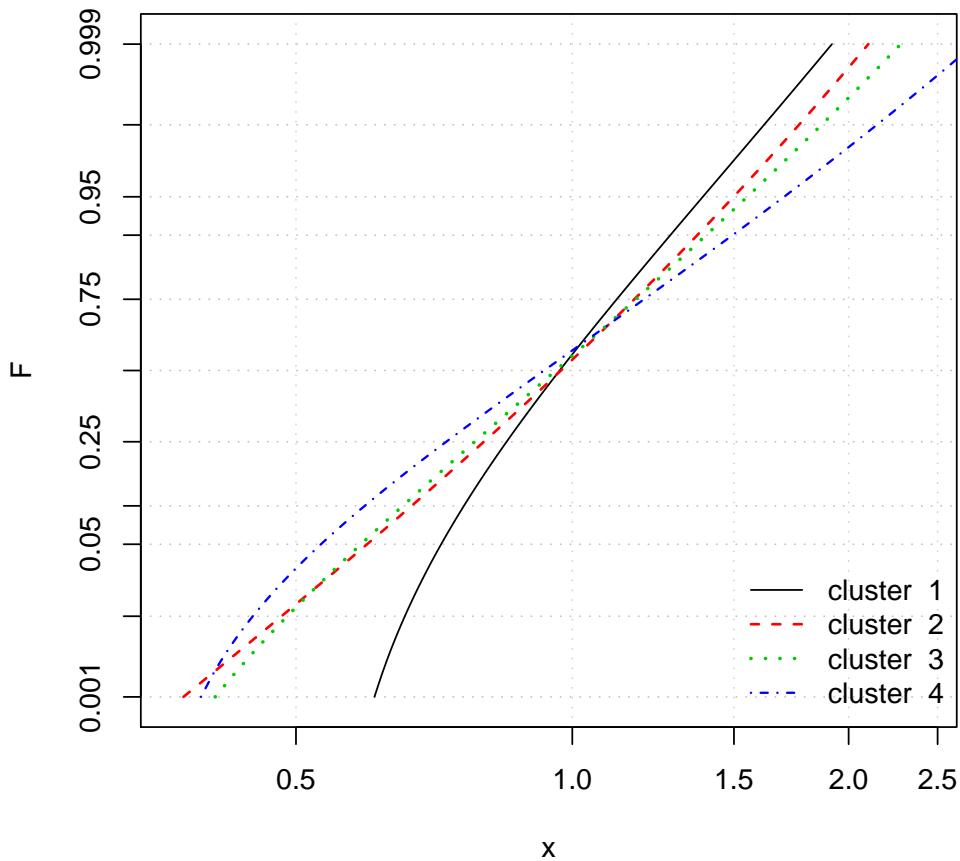


Comparison between regional growth-curves:

```

> spess = c(1, 1.5, 2, 1.3)
> Fs <- seq(0.001, 0.999, by = 0.001)
> lognormplot(D15adim, line = FALSE, type = "n", )
> for (i in 1:nclusters) {
+   qq <- invF.gamma(Fs, paramgamma[[i]]$xi, paramgamma[[i]]$beta,
+                      paramgamma[[i]]$alfa)
+   normpoints(qq, type = "l", lty = i, col = i, lwd = spess[i])
+ }
> legend("bottomright", paste("cluster ", c(1:nclusters)), col = c(1:nclusters),
+        lty = c(1:nclusters), lwd = spess, bty = "n")

```



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