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ON SEQUENTIAL ESTIMATION OF THE LARGEST NORMAL MEAN WHEN THE VARIANCE IS KNOWN*

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SUMMARY. We define a class of stopping times for estimating the largest of k normal means when the variance is known. The class can achieve significant reduction in sample size compared to a related procedure due to Blumenthal (1976) because it employs an elimination feature which halts sampling on populations furnishing no information about the largest mean. The asymptotic behaviour of the stopping times and the mean square consistency of the estimators are studied.

1. Introduction

Let $\theta_1, \ldots, \theta_k$ be the unknown means of k normal populations with common known variance σ^2 (henceforth taken as unity). Let $\bar{X}_{1n}, \ldots, \bar{X}_{kn}$ be the sample means of n observations taken from the k populations, and define the ordered population and sample means by $\theta_{[1]} \leqslant \ldots \leqslant \theta_{[k]}$ and $\bar{X}_{[1]n} \leqslant \ldots \leqslant X_{[k]n}$. The problem is to construct a sequential stopping time for the estimation of $\theta_{[k]}$ (by an estimator θ_n^*, often takes as $\bar{X}_{[k]n}$) with prespecified mean square error (MSE) r. The procedures we investigate depend on the estimates $\Delta_{in} = \bar{X}_{[k]n} - \bar{X}_{[i]n}$ of $\Delta_i = \theta_{[k]} - \theta_{[i]}$ $(i = 1, \ldots, k)$.

Blumenthal (1976) constructed a purely sequential stopping time N_B and a related two-stage procedure N_B^*, obtaining results which may be summarized as follows. For $\Delta_1, \ldots, \Delta_{k-1}$ fixed, rN_B and rN_B^* have almost sure limits as $r \to 0$, but asymptotic mean square consistency is verified only for the two-stage procedure for $k = 2$. If each Δ_i is proportional to $r^{1/2}(\Delta_i \sim r^{1/2})$, neither N_B nor N_B^* has an almost sure limit, but the limit distribution is computed only for the two-stage procedure N_B^* when $k = 2$; asymptotic mean square consistency is not checked in this case for the sequential procedure N_B. Blumenthal indicates that for $k = 2$, his procedures will achieve approximately 10% savings in sample size when compared to a conservative, fixed-sample procedure.

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