MVN package: Multivariate Normality Tests

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Abstract

Assessing the assumption of multivariate normality is required by many parametric multivariate statistical methods, such as discriminant analysis, principal component analysis, MANOVA, etc. Here, we present an R package to asses multivariate normality. The `MVN` package contains three most widely used multivariate normality tests, including Mardia's, Henze-Zirkler's and Royston's multivariate normality tests.

Contents

1 Preparation of input data

`MVN` package expects a numeric matrix or a data frame that contains minimum two variables. In this vignette, we will work with the *Iris* data set. This data set is a multivariate data set introduced by Ronald A. Fisher (1936) as an application of discriminant analysis [1]. It is also called Anderson's Iris data set because Edgar Anderson collected the data to measure the morphologic variation of Iris flowers of three related species [2]. The data set consists of 50 samples from each of three species of Iris including *setosa*, *virginica* and *versicolor*. For each sample, four variables were measured including the length and the width of the *sepals* and *petals*, in centimeters. We will check the multivariate normality of the *Iris* data set by using three multivariate normality tests, including Mardia's, Royston's and Henze-Zirkler's multivariate normality tests.

First, we can call our data set using `data` function and display it using `head` function as follows:

```
data(iris)
head(iris)

##   Sepal.Length Sepal.Width Petal.Length Petal.Width Species
## 1          5.1         3.5          1.4         0.2  setosa
## 2          4.9         3.0          1.4         0.2  setosa
## 3          4.7         3.2          1.3         0.2  setosa
## 4          4.6         3.1          1.5         0.2  setosa
## 5          5.0         3.6          1.4         0.2  setosa
## 6          5.4         3.9          1.7         0.4  setosa
```

The *Iris* data is in `data.frame` format which consists of 5 variables (*Sepal.Length*, *Sepal.Width*, *Petal.Length*, *Petal.Width*, and *Species*) and 150 samples.

```
class(iris)

## [1] "data.frame"

dim(iris)

## [1] 150   5
```

For simplicity, we will work with a subset of the *Iris* data with first 50 samples without class label.

```
Iris = iris[1:50, 1:4]
head(Iris)

##   Sepal.Length Sepal.Width Petal.Length Petal.Width
## 1          5.1         3.5          1.4         0.2
## 2          4.9         3.0          1.4         0.2
## 3          4.7         3.2          1.3         0.2
## 4          4.6         3.1          1.5         0.2
## 5          5.0         3.6          1.4         0.2
## 6          5.4         3.9          1.7         0.4
```

2 Multivariate Normality Tests

We will introduce three multivariate normality tests below, including Mardia's, Henze-Zirkler's and Royston's Multivariate Normality Tests.

Before using our multivariate normality tests, we need to load our `MVN` package as follows:

```
library(MVN)
```

2.1 Mardia's Multivariate Normality Test

Mardia's test is based on multivariate extensions of *skewness* and *kurtosis* measures [3]. Now, we will check the multivariate normality of the *Iris* data using `mardiaTest` function in the `MVN` package. This function calculates the Mardia's multivariate skewness and kurtosis coefficients as well as their corresponding statistical tests. For large sample size the multivariate skewness is asymptotically distributed as a chi-square random variable; here it is corrected for small sample size. Likewise, the multivariate kurtosis is distributed as a unit-normal [4–6].

```
result <- mardiaTest(Iris, cov = TRUE, qqplot = FALSE)
result

##    Mardia's Multivariate Normality Test
## ---------------------------------------
##    data : Iris
##
##    g1p            : 3.08
##    skew           : 25.66
##    p.value.skew   : 0.1772
##
##    g2p            : 26.54
##    kurtosis       : 1.295
##    p.value.kurt   : 0.1953
##
##    small.skew     : 27.86
##    p.value.small  : 0.1128
## ---------------------------------------
```

Here, `g1p`: Mardia's estimation of multivariate skew, `skew`: Mardia's skew statistic, `p.value.skew`: p-value of skew statistic, `g2p`: Mardia's g2p estimate of multivariate kurtosis, `kurtosis`: Mardia's multivariate kurtosis statistic, `p.value.kurt`: p-value of kurtosis statistic, `small.skew`: Mardia's small sample skew statistic and `p.value.small`: p-value of small sample skew statistic.

As seen above results, both skewness ($p = 0.1772$) and kurtosis ($p = 0.1953$) values indicate multivariate normality.

2.2 Henze-Zirkler's Multivariate Normality Test

The Henze-Zirkler test is based on a non-negative functional distance that measures the distance between two distribution functions. If the data is multivariate normal, the test statistic is approximately log-normally distributed. It proceeds to calculate the mean, variance and smoothness parameter. Then, mean and variance are log-normalized and the p-value is estimated. We can use `hzTest` function in the `MVN` package to calculate the Henze-Zirkler's Multivariate Normality Test [7–11].

```
result <- hzTest(Iris, cov = TRUE, qqplot = FALSE)
result

##   Henze-Zirkler's Multivariate Normality Test
## ---------------------------------------------
##   data : Iris
##
##   HZ      : 0.9488
##   p-value : 0.04995
## ---------------------------------------------
```

Here, `HZ` is the value of Henze-Zirkler statistic at significance level 0.05 and `p-value` is a p-value for the Henze-Zirkler's Multivariate Normality Test.

Since the p-value, which optain from the `hzTest`, lower than 0.05, one can conclude that this multivariate data set deviates from multivariate normality.

2.3 Royston's Multivariate Normality Test

Royston's H test uses `Shapiro-Wilk's W` statistic for multivariate normality. However, if kurtosis of the data greater than 3 then `Shapiro-Francia` test is used for leptokurtic samples else `Shapiro-Wilk` test is used for platykurtic samples [10, 12–18].

```
result <- roystonTest(Iris, qqplot = FALSE)
result

##   Royston's Multivariate Normality Test
## ---------------------------------------------
##   data : Iris
##
##   H       : 31.52
##   p-value : 2.188e-06
## ---------------------------------------------
```

Here, `H` is the value of Royston's H statistic at significance level `0.05` and `p-value` is an approximate p-value for the test with respect to equivalent degrees of freedom (`edf`).

According to the Royston's Multivariate Normality Test, the *Iris* data set does not appear to follow a multivariate normal distribution ($p < 0.001$).

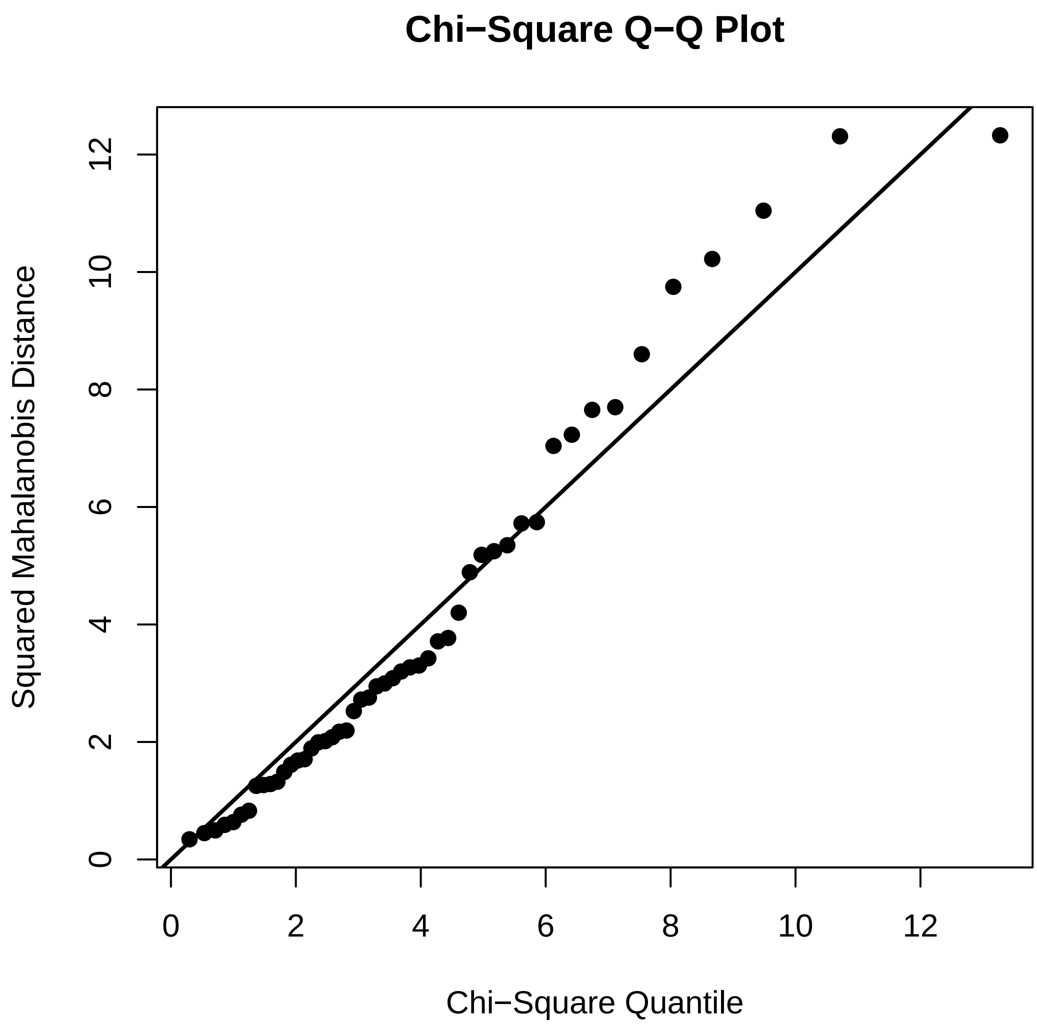
3 Multivariate Normality Plots

Our `MVN` package has ability to create three multivariate plots. We can use `qqplot = TRUE` option in the `mardiaTest`, `hzTest` and `roystonTest` functions to create a chi-suqare Q-Q plot. Furthermore, we can use `mvnPlot` function in our `MVN` package to create perspective and contour plots for binary data sets.

3.1 Q-Q Plot

We can create a chi-square Q-Q plot for our *Iris* data set to see whether there is any deviation from multivariate normality.

```
result <- roystonTest(Iris, qqplot = TRUE)
```



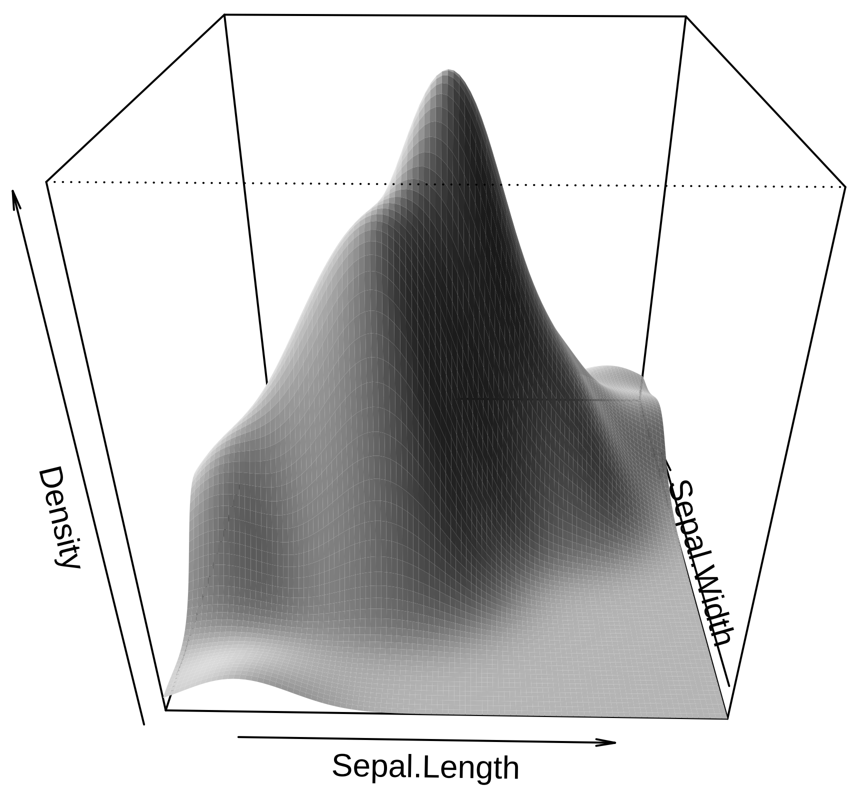

```
result

##   Royston's Multivariate Normality Test
## ---------------------------------------------
##   data : Iris
##
##   H       : 31.52
##   p-value : 2.188e-06
## ---------------------------------------------
```

If the data set follows approximately a multivariate normal distribution, the resulting plot should be roughly straight line. As you can see from the chi-square Q-Q plot above, there are some deviations from the straight line and this indicates possible departures from a multivariate normal distribution.

3.2 Perspective and Contour Plots

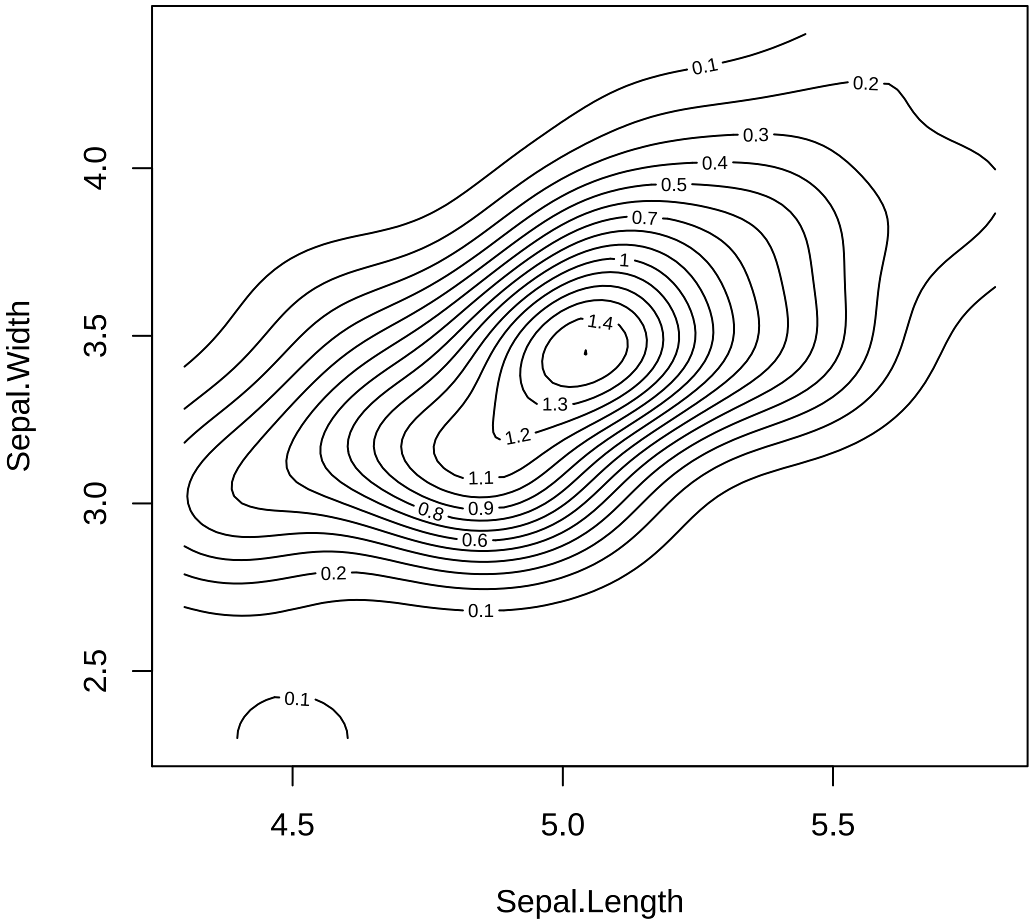
We can use the `mvnPlot` function in the `MVN` package to create a perspective plot for a binary data set. In order to get a perspective plot, we should continue with two variables, i.e., bivariate normal distribution. As an example, we subset first 50 rows and *sepal* measures of *Iris* data. Sepal measures of first 50 samples are bivariate normal. We can see that from the perspective plot. Perspective plot produces 3-dimensional bell-shaped graph when data is bivariate normal.

```
Iris = iris[1:50, 1:2]
result = hzTest(Iris)
mvnPlot(result, type = "persp", default = TRUE)
```



Another alternative is to use 2-dimensional contour graphs. We can use the `mvnPlot` function in the `MVN` package to create a contour plot for a binary data set. Contour graphs are very useful since it gives information about normality and correlation at the same time. From contour graph below, we can say that there is a positive correlation among *sepal* measures of flowers since contour lines lie around main diagonal.

```
mvnPlot(result, type = "contour", default = TRUE)
```



4 Session info

```
sessionInfo()

## R version 3.0.3 (2014-03-06)
## Platform: x86_64-apple-darwin10.8.0 (64-bit)
##
## locale:
## [1] C/tr_TR.UTF-8/tr_TR.UTF-8/C/tr_TR.UTF-8/tr_TR.UTF-8
##
## attached base packages:
## [1] stats     graphics  grDevices utils     datasets  methods   base
##
## other attached packages:
## [1] MVN_3.1       MASS_7.3-29   moments_0.13  nortest_1.0-2 knitr_1.5
##
## loaded via a namespace (and not attached):
## [1] codetools_0.2-8 digest_0.6.4    evaluate_0.5.1  formatR_0.10
## [5] highr_0.3       stringr_0.6.2   tools_3.0.3
```

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